OPTIMAL REACTIVE POWER PLANNING AND PRICING ANALYSIS IN A COMPETITIVE ELECTRICITY MARKET

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By

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To

My Teachers & Family Members

CERTIFICATE

This is to certify that the work contained in this thesis entitled ***"Optimal Reactive Power Planning and Pricing Analysis in a Competitive Electricity Market"***, has been carried out by **Tukarama Moger (Y3104104)** under my supervision and that this work has not been submitted elsewhere for a degree.

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Abstract

A methodology for reactive power planning and pricing analysis is presented. Attention is given to the reactive power marginal prices in a competitive electricity market. The methodology has been implemented using a modified optimal power flow. The planning problem involves optimal placement and sizing of capacitor at load buses to improve the system voltage profiles and reduce losses in a network so that operating and investment costs are minimum. A simple bus-wise cost benefit analysis (CBA) is presented which involves solving a modified OPF problem iteratively. The CBA incorporates detailed hourly loading conditions at a bus and achieves a fairly accurate estimate of the benefits from capacitor placement. A reactive power marginal price is studied in details under different system operating conditions to observe how these conditions influence reactive power marginal prices. The IEEE-14 and IEEE-118 bus systems have been used for the application of methodology. Results demonstrate that the active and reactive power marginal prices give economic signals that could impel even more the participation of agents of competitive reactive power markets.

Chapter 1

Introduction

1.1 General

The traditional electric power industries, worldwide, have been operated as regulated industries, where utilities typically own generation, transmission and distribution over a wide geographical area. Such industries are known as vertically integrated electric industries. Regulation means that the government has set down laws and rules that put limits on and define how a particular industry can operate. It provides guidelines for industries to do business practices and operate their facilities within recommended safety guidelines. Historically, the need to regulate the electricity market was to meet the following points:

- Attract the investors to develop the electricity network.
- Increase the popularity of electricity usage.
- Develop risk free environment for investment.
- Distribution of electricity in non-discriminatory fashion.

The salient features of the regulated electric utility are as following [1]:

- Monopoly franchise: In an area, only one utility is allowed to generate, transmit and sell electric power to consumers.
- Obligation to serve: It must provide electricity to all consumers and not only to those that would be profitable.

- Regulatory requirement: Business and operating practices must conform to the guidelines and rules set down by the government.
- Least cost operation: The utility must operate in a manner to minimize its overall revenue requirement.
- Regulated rates: tariff has to be fixed in accordance with the government regulatory rules and guidelines.
- Assured rate of return: The utility is assured a 'fair' return on its investment, if it is follows the regulatory rules and guidelines.

1.1.1 De-regulation in Power Industry

Beginning in the 1980's, the electric utility industries, worldwide, are undergoing changes due to their restructuring and de-regulation. It is hoped to increase system efficiencies and improve benefits to electricity consumers. The need to go for the de-regulation is due to the following reasons:

- Regulation is no longer necessary.
- By introducing competition, electricity prices may drop.
- Much wider customer choice and more attention to improved services.
- Establishment of competition marketplace for electricity products and services results in the new technological innovations.

Attitude towards de-regulation (or more appropriate re-regulation) varies between countries. Many countries are adapting a wait and see attitude, some are performing investigations, and few others are already moved to restructure their energy market place. Several countries have already moved towards de-regulation such as New Zealand, Chile, Norway, Sweden, Australia, United Kingdom (U. K.) and Argentine.

The market system will consist of several companies such as generation companies (GENCOs), distribution companies (DISCOs), transmission companies (TRANSCOs), energy service companies (ESCOs), ancillary services companies (ANCILCOs), independent system operator (ISO), regional transmission groups (RTGs) and national

electricity regulatory commission. The principal characteristics of a competitive structure are the identification and separation of various tasks, which are normally carried out, in the traditional system. The purpose is to open them to competition, whenever found practical and profitable. This process is also called as 'unbundling'.

With the new open access to the consumer base, it becomes possible for independent power producers (IPPs) to install highly efficient, low cost power plants. The unbundling of utility services may results in more equitable tariffs for individual task. The single tariff used presently averages cost of many services. The new tariff may more closely reflect the actual worth of the unbundled services.

It is possible to split up, generation into sufficiently large number of smaller independent competing generating companies or GENCOs. IPPs are also allowed to compete at this level. This unbundling is not possible in transmission network. If the transmission network is split into different parts and then sold to a number of companies, it is difficult to devise mechanism for TRANSCOs to compete fairly. The reason is that power flowing due to a contract between a generator and a load can not be guaranteed to flow through a specific TRANSCO instead it will flow through the entire network, following the network physical laws. Due to this reason, TRANSCO, generally, has monopoly and owned by a company independent form the market players.

The main role of ISO is to coordinate the various services purchased in the open market. ISO implements its actions according to a set of rules, previously agreed upon by all market participants. It has to ensure that total generation meets total demand. Its major responsibilities include system security, transmission pricing and open access. ISO can no longer be held responsible for supplying total demand most efficiently. If the price of the electricity is very high and some consumers can't pay for it, ISO can't be held responsible for supplying such load unconditionally [2]. The body which implements the mechanism by which all market participants trade electricity, is called power exchange or PX. The ISO and PX could be same (as in U.K.) or separate (as in California) bodies.

Price stability is the advantage of regulated power industry. Both buyers and sellers know the price of power in advance and can depend on the prices to be stable. Whereas, in a competitive marketplace prices fluctuate unpredictably depending on the market conditions. This price volatility makes the planning of energy resources more of a challenge to the users.

1.1.2 Literature Review of Reactive Power Markets

During the era of vertically integrated utilities, reactive power was viewed strictly as an engineering issue, something to be built sufficiently into the system in a centralized way. Studies conducted before restructuring focused on whether reactive power and other alternating current characteristics were being represented correctly in engineering calculations, such as contingency models and distribution factors [4]. Other engineering research was beginning to explore the role reactive power controls had in increasing the functioning of the grid, perhaps in obviating (or at least postponing) some system upgrades to generation and transmission [5].

Beginning in the early 1980's, researchers were beginning to think about alternate ways of pricing electricity to achieve specific objective, such as maximal social welfare or system reliability. Caramanis et al. [6] presented a new concept in electricity pricing, a method which evolved into what is now known as locational pricing. In this work, it was suggested that a market for electricity can efficiently set location-specific prices based on instantaneous supply and demand that promote consumption patterns that benefit the transmission system. Implementing separate prices for real and reactive power would produce the most efficient pricing outcomes, even though the authors assert that the price of reactive power will often be insignificant compared to real power prices. The calculations for real power from this work were expanded upon by Schweppe et al [3]. However, this later work does not discuss reactive power. Although some of the early economic publications mention reactive power or various schemes for voltage control, and indicate separate prices might be desirable, none rigorously consider the implications of reactive power prices or mechanisms for setting these prices [7].

In the early 1990s, with the restructuring of the industry eminent, researchers began looking more seriously at pricing both real and reactive power in an economically efficient way. The new emphasis on markets for electricity created a new focus on reactive power pricing in the literature: whether it was important, how it should be done, what would be the resulting prices. Baughman and Siddiqi [8] presented an early argument that because the physics of real and reactive power are so closely tied, simultaneous pricing of real and reactive power would be important to the development of electricity markets and that in the presence of voltage constraints, reactive power prices can be extremely high.

After the lot of debate, it became accepted that reactive power management had to adapt in this new, restructured era. Now in addition to effecting grid security, reactive power and voltage control played a part in determining market efficiency [9]. Much of the research that followed focused on alternative ways to manage and dispatch reactive power in the future, including whether desirable system voltage profiles exist and how could they be determined. There was also increased production of technical and non technical [10] papers discussing reactive power, presumably because a wider audience wider than just the engineering community needed to know what it was, how it worked and why it was important. Kirby and Hirst [11] discussed the role of transmission in voltage control, characteristics of voltage control equipment and strategies for voltage control management. Several years later another report came out that described reactive power balance, reactive power and transmission and black-start techniques, generator reactive capability (comparing actual performance with manufacturer name plate characteristics) and the importance of optimizing transformer tap positions for producing and absorbing reactive power [12]. Alvarado et al. produced a comprehensive literature and market review of reactive power pricing strategies [13] and made a determination of how to handle reactive power in system.

In looking to the future of markets and policy, two general areas of research developed. One examines decentralized incentives for reactive power capacity and dispatch, how optimal power flows (OPFs) can be modified to incorporate reactive power costs, and what price signals would best capture the incentives for building capacity and ensuring

performance. The other focuses on the role of centralized planning and control of reactive power planning and production in the era of restructured electricity markets.

1.1.3 Reactive Power Planning and Pricing

What is reactive power? Almost all bulk electric power is generated, transmitted and consumed in alternating current (AC) networks. Elements of AC systems supply (or produce) and consume (or absorb or lose) two kinds of power: real power and reactive. Real power accomplishes useful work (e.g., runs motors and light lamps). Reactive power supports the voltages that must be controlled for system reliability [10].

Voltage control in an electric power system is important for proper operation of electric power equipment to prevent damage such as overheating of generators and motors, to reduce transmission losses and to maintain the ability of the system to withstand disturbances and prevent voltage collapse. Hence reactive power supply is essential for reliably operating the electric transmission system. Inadequate reactive power has led to voltage collapses and has been a major cause of several major power outages worldwide. Not only is reactive power necessary to operate the transmission system reliably, but it can also substantially improve the efficiency with which real power is delivered to customers.

Reactive power is required by two types of customers. First and primarily, reactive power is needed by the system operator to maintain voltage levels and ensure the reliability of the transmission network. Second, reactive power is supplied and consumed in varying amounts by most market participants. The combination of the system operator's need for maintaining system reliability and reactive power consumption by real power load determines the total system reactive power needs.

Reactive power is difficult to transmit. At high loadings, relative losses of reactive power on transmission lines are often significantly greater than relative real power losses. Reactive power consumption or losses can increase significantly with the distance transmitted. Losses in transmission lead to the expression that reactive power does not travel well. When there is not enough reactive power supplied locally, it must be supplied

remotely, causing larger currents and voltage drops along the path. This factor limits the geographic scope of the reactive power market and, thus, the number of suppliers that can provide reactive power. Thus, reactive power usually must be procured from suppliers near where it is needed. Increasing reactive power production at certain locations (usually near a load center) can sometimes alleviate transmission constraints and allow cheaper real power to be delivered into a load pocket.

Reactive power needs are a critical part of the planning process and to be managed or compensated in a way to ensure sufficient amounts are being produced to meet demand and so that the electric power system can run efficiently. Significant problems (e.g., abnormal voltages and system instability) can occur if reactive power is not properly managed. Capacitors, which supply reactive power, can be switched into a system in real-time to compensate for the reactive power consumed by the electric power system during periods of heavy loading. Similarly, inductors, which consume reactive power, are added to compensate for the reactive power supplied by the electric power system during periods of light loading. These devices are installed throughout the electric power system to maintain an acceptable voltage profile for a secure and efficient power system operation.

After the reliability needs of the system have been determined, the goal of reactive power pricing and procurement should be to encourage two efficient outcomes. First, it should encourage efficient and reliable investment in the infrastructure needed to maintain the reliability of the transmission system. Second, it should provide incentives for the reliable and efficient production and consumption of reactive power from the existing available infrastructure. Additionally, it is important that any pricing system allows the system operator real-time control over reactive power resources. While pricing rules should complement the reliability needs of the system, in some situations, the system operator may need to adjust reactive power resources, applying out of market dispatch instructions during system emergencies.

One of the key characteristics of competitive markets is that the prices in competitive markets tend to reflect sellers' marginal costs. Such prices send desirable signals to

market participants. This characteristic helps ensure that the efficient amount of output is produced and consumed.

1.2 Objectives of the Thesis

The main objectives of the present study have been the following:

- To develop a modified optimal power flow model for reactive power planning considering minimization of system generating cost and the cost of adding new capacitors as an objective to determine the minimum investment in reactive power sources which are necessary to improve the system voltage profiles and reduces losses in a network. So that proper degree of reliable operation of the power system can be achieved.

- The optimal placement and size of capacitors at load buses is obtained from cost benefit analysis (CBA). The CBA incorporates detailed hourly loading conditions at a bus and achieves a fairly accurate estimate of the benefits from capacitor placement. Therefore, the utilities can reduce their capital investment on reactive power sources and same time ensuring the reliable operation of the power system (PS) and also get the maximum benefit from the limited reactive power sources.

- It is realized that establishing an accurate pricing structure of reactive power can not only recover the costs of reactive power providers, but also provide economic information for real-time operation. A reactive power pricing analysis is carried out to observe the impact of different system operating conditions including change of objective function, load power factor, daily load fluctuations and voltage control on reactive power marginal prices. This is very much important in competitive electricity market.

1.3 Thesis Organization

The work carried out under this thesis has been presented in four chapters.

Chapter 1 introduces an overall perspective of the ongoing deregulation and restructuring of power system industries. It presents the relevant literature review of reactive power markets. It also highlights few technical & economic issues in the reactive power planning & pricing in electricity industry.

Chapter 2 presents the formulation of modified optimal power flow model for reactive power planning to determine the minimum investment in reactive power sources with an objective function to minimize the system generating cost and the cost of adding new capacitors. An explicit bus-wise cost benefit analysis (CBA) is carried out to decide upon the optimal placement of capacitors and their sizes are discussed.

Chapter 3 presents the analysis of reactive power pricing by considering the impacts of change in objective function and different system operating conditions including load power factor, daily load fluctuation and voltage control on reactive power marginal prices and active power marginal prices are highlighted.

The IEEE-14 and IEEE-118 bus systems have been considered for the detail case studies for reactive power planning and pricing analysis in Chapter 2 and Chapter 3, respectively.

Finally, *Chapter 4* concludes the main findings of the work presented in this thesis and identifies scope for future work in this area.

Chapter 2

Optimal Reactive Power Planning

2.1 Introduction

Power system reactive power planning has long been an important issue for the power industry. As the trend towards open access and deregulation occurred in the power industry in recent years, power systems are becoming more and more stressed and stability problems are brought for more attention. One of the problems, which cause voltage instability, is insufficient reactive power supply. In the electricity market, the system operator must make sure there are appropriate reactive reserves planning for the system in order to maintain voltage profiles. Properly planned reactive reserve minimizes the risk of voltage collapse or low voltages following contingencies as well as reducing transmission losses by keeping appropriate voltage profiles.

To maintain security and adequacy in a bulk power system, FERC's Order No. 888 in 1996 specified six services called ancillary services that the electrical transmission providers must provide. These six services constitute system control, reactive supply and voltage control, regulation, operating spinning reserve, operating supplemental reserve, and the energy imbalance. One of the key ancillary services is the Reactive Supply (VARs) and Voltage Control [11, 13-14].

The goal of reactive power planning is to determine the minimum investment in reactive power sources which are necessary to correct unacceptable voltage profiles during anticipated normal and contingency conditions. The reactive power planning is an

optimization problem. Solving this problem requires finding an optimal solution that minimizes an objective function while satisfying the constraints.

The reactive power-planning (RPP) problem involves optimal allocation and sizing of reactive power sources at load buses to improve the system voltage profile and reduces losses such that the investment costs as well as operating costs are minimum. The criteria used for RPP have been to minimize total operating cost of the system and the cost of new reactive power sources (capacitors). A simple bus-wise cost-benefit analysis (CBA) scheme has been presented in details to estimate the benefits from capacitor placement, which involves solving modified optimal power flow problem (OPF) literately. The CBA incorporates detailed hourly loading conditions at a bus and achieves a fairly accurate estimate of the benefits from capacitor placement. The flow chart for this is shown in Fig. 2.1.

In this chapter a nonlinear optimization problem has been formulated. A program has been developed for the solution of the OPF problem using constrained non-linear optimization function in MATLAB's Optimization Toolbox [16] with the help of MatPower package [17]. The OPF solution is achieved when the tolerance on per unit real and reactive power mismatch is 10^{-8}. The study has been carried out on IEEE-14 and IEEE-118 bus system [18]. The formulation of OPF problem described below.

2.2 An OPF Model for Optimal Allocation & Sizing of Capacitors

A modified OPF formulation used for allocation and sizing of capacitors on the load buses discussed in this chapter. These additional sources are required to provide the necessary reactive power support at load buses, more so, during the peak loads. OPF is computed for every hour of the load curve.

The modified OPF objective function comprises the aggregate cost of generation and cost of adding new capacitors. If system parameters viz, voltage, line flows etc. are within

their specified limits, the utility would prefer to operate on a least-cost schedule which may leads to somewhat higher losses would be incurred since cheaper generating sources located far away from the load centers. However, it may be worthwhile for the utility to bear these additional losses and operate at minimum system cost rather than switching generation to relatively expensive units located near to the load centers and reduces the losses.

2.2.1 Objective Function

The Modified Optimal Power Flow (MOPF) may be formulated as a non-linear programming problem to minimize a objective function,

$$COST = \sum_{i \in NG} C_i(P_{gi}) + \sum_{i \in NL} Q_{ci}.CAPCOST \qquad \$/Hr \tag{2.1}$$

Where,

COST: Objective function ($/Hr)

CAPCOST: Cost of installing new capacitor ($/MVAr-hr)

C_i(.): Generator i^{th} cost characteristic

NG: No. of generating buses

NL: No. of load buses

The generators, in this work, are assumed to have quadratic cost characteristics which is given by,

$$C_i\left(P_{gi}\right) = a_i P_{gi}^2 + b_i P_{gi} + c_i \qquad \$/Hr \tag{2.2}$$

Where

a_i, b_i, and c_i are cost coefficients pertaining to the i^{th} generator.

The objective of pricing policy is to minimize the production cost of electricity. The various constraints to be fulfilled during optimization are as follows,

2.2.2 Operating Constraints

2.2.2.1 Load Flow Equations

A set of equations that characterizes the balance of real and reactive powers at each node in the system is given by,

$$P_{gi} - P_{di} = \sum_{j=1}^{N} V_i V_j Y_{ij} \, Cos\left(\delta_i - \delta_j - \theta_{ij}\right) \tag{2.3}$$

For i =1 ....N

$$Q_{gi} - Q_{di} = \sum_{j=1}^{N} V_i V_j Y_{ij} \, Sin\left(\delta_i - \delta_j - \theta_{ij}\right) \tag{2.4}$$

For i =1 ....NG

$$Q_{gi} - Q_{di} + Q_{ci} = \sum_{j=1}^{N} V_i V_j Y_{ij} \, Sin\left(\delta_i - \delta_j - \theta_{ij}\right) \tag{2.5}$$

For i = 1....NL

Where,

N: Total no. of buses

i, j: Index for buses

P_{gi}: Real power generation at i^{th} bus

Q_{gi}: Reactive power generation at i^{th} bus

P_{di}: Real power demand at i^{th} bus

Q_{di}: Reactive power demand at i^{th} bus

V_i: Voltage magnitude at i^{th} bus

Y_{ij}: Element of network admittance matrix [Y_{BUS}]

θ_{ij}: Phase angle of Y_{ij}

δ_i: Voltage angle at i^{th} bus

2.2.2.2 Generation Limits

The generating plants of the utility have a maximum generating capacity, above which it is not feasible to generate due to technical or economic reasons. Generation limits are important in determining the operating points and marginal costs of generation. Generation limits are usually expressed as maximum and minimum active power and reactive power outputs,

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \tag{2.6}$$
$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \tag{2.7}$$
For i = 1….NG

Where,

P_{gi}^{min}, P_{gi}^{max}: Real power generation limits at i^{th} bus

Q_{gi}^{min}, Q_{gi}^{max}: Reactive power generation limits at i^{th} bus

2.2.2.3 Voltage Limits

Voltage limits refer to the requirement for the system bus voltages to remain within a narrow range of levels. Since voltages are affected primarily by reactive power flows, the marginal cost of supplying reactive power to a bus is directly dependent in the voltage level requirement at that bus. The voltage limits can be expressed by the following constraints.

$$\begin{aligned} &|V_i| = Const. \qquad for\ i = 1, \ldots NG \\ &|V^{\min}| \leq |V_i| \leq |V^{\max}| \quad for\ i = 1, \ldots NL \end{aligned} \tag{2.8}$$

Where,

V_i^{min}, V_i^{max}: Limits on i^{th} bus voltage levels

2.2.2.4 Transmission Limits

Transmission limits refer to the maximum power or current that a given transmission line is capable of transmitting under given conditions. These limits based on thermal considerations or stability considerations. Thermal limits usually dominate for shorter lines. Dynamic stability limits dominate longer lines. Transmission limits are expressed in terms of the maximum active power flow through the lines.

$$P_{ij} \leq P_{T\max} \qquad for\ i \neq j, \quad i, j\ \varepsilon\ Nl \tag{2.9}$$

Where,

P_{ij}: Active power transfer from i^{th} bus to j^{th} bus

P_T^{max}: Max power transfer limit

Nl: No. of links (or lines)

2.2.2.5 Reactive Power (VAr) Injection Limits

$$Q_{ci} \leq Q_c^{\max} \qquad \textit{for } i = 1, \ldots NL \tag{2.10}$$

Where,

Q_{ci}: Reactive power support from new capacitor at i^{th} bus

Q_c^{max}: Max reactive power support possible to add

Based on the above mathematical model the corresponding Lagragian function of the optimization problem to be minimized over all active power generation levels P_g, generators reactive power generation levels Q_g, capacitors reactive power generation level Q_c, Voltage levels V, and voltage angle δ, is;

$$\begin{aligned}
L(P_g, Q_g, Q_c, V, \delta) = & \sum_{i \in NG} C_i(P_{gi}) + \sum_{i \in NL} Q_{ci} \,.\, CAPCOST - \sum_{i \in N} (MC_{pi}) \, [P_{gi} - P_{di} - \\
& \sum_{j \in N} V_i V_j Y_{ij} \, Cos(\delta_i - \delta_j - \theta_{ij})] - \sum_{i \in NG} (MC_{qi}) \, [Q_{gi} - Q_{di} - \sum_{j=1}^{N} V_i V_j Y_{ij} \, Sin(\delta_i - \delta_j - \theta_{ij})] \\
& - \sum_{i \in NL} (MC_{qi}) \, [Q_{gi} - Q_{di} + Q_{ci} - \sum_{j \in N} V_i V_j Y_{ij} \, Sin(\delta_i - \delta_j - \theta_{ij})] - \sum_{i \in NL} \nu_{i,\min} (V_i - V_i^{\min}) \\
& + \sum_{i \in NL} \nu_{i,\max} (V_i - V_i^{\max}) - \sum_{i \in NG} \lambda_{i,\min} (P_{gi} - P_{gi}^{\min}) + \sum_{i \in NG} \lambda_{i,\max} (P_{gi} - P_{gi}^{\max}) \\
& - \sum_{i \in NG} \mu_{i,\min} (Q_{gi} - Q_{gi}^{\min}) + \sum_{i \in NG} \mu_{i,\max} (Q_{gi} - Q_{gi}^{\max}) - \sum_{i \in NL} \sigma_{i,\min} (Q_{ci} - Q_c^{\min}) \\
& + \sum_{i \in NL} \sigma_{i,\max} (Q_{ci} - Q_c^{\max}) + \sum_{i \in N} \sum_{j \in N}^{j \neq i} \eta_{ij} (|P_{ij}| - P_T^{\max})
\end{aligned} \tag{2.11}$$

Where,

L: Lagragian function

MC_{pi}: Lagrange multiplier on the active power equation at i^{th} bus

MC_{qi}: Lagrange multiplier on the reactive power equation at i^{th} bus

$\lambda_{i,min}$: Lagrange multiplier on the min. active power generation limit at i^{th} bus

$\lambda_{i,max}$: Lagrange multiplier on the max. active power generation limit at i^{th} bus

$\mu_{i,min}$: Lagrange multiplier on the min. reactive power generation limit at i^{th} bus

$\mu_{i,max}$: Lagrange multiplier on the max. reactive power generation limit at i^{th} bus

η_{ij}: Lagrange multiplier on the active power flow limit from i^{th} bus to j^{th} bus

$\nu_{i,min}$: Lagrange multiplier on the min. voltage level at i^{th} bus

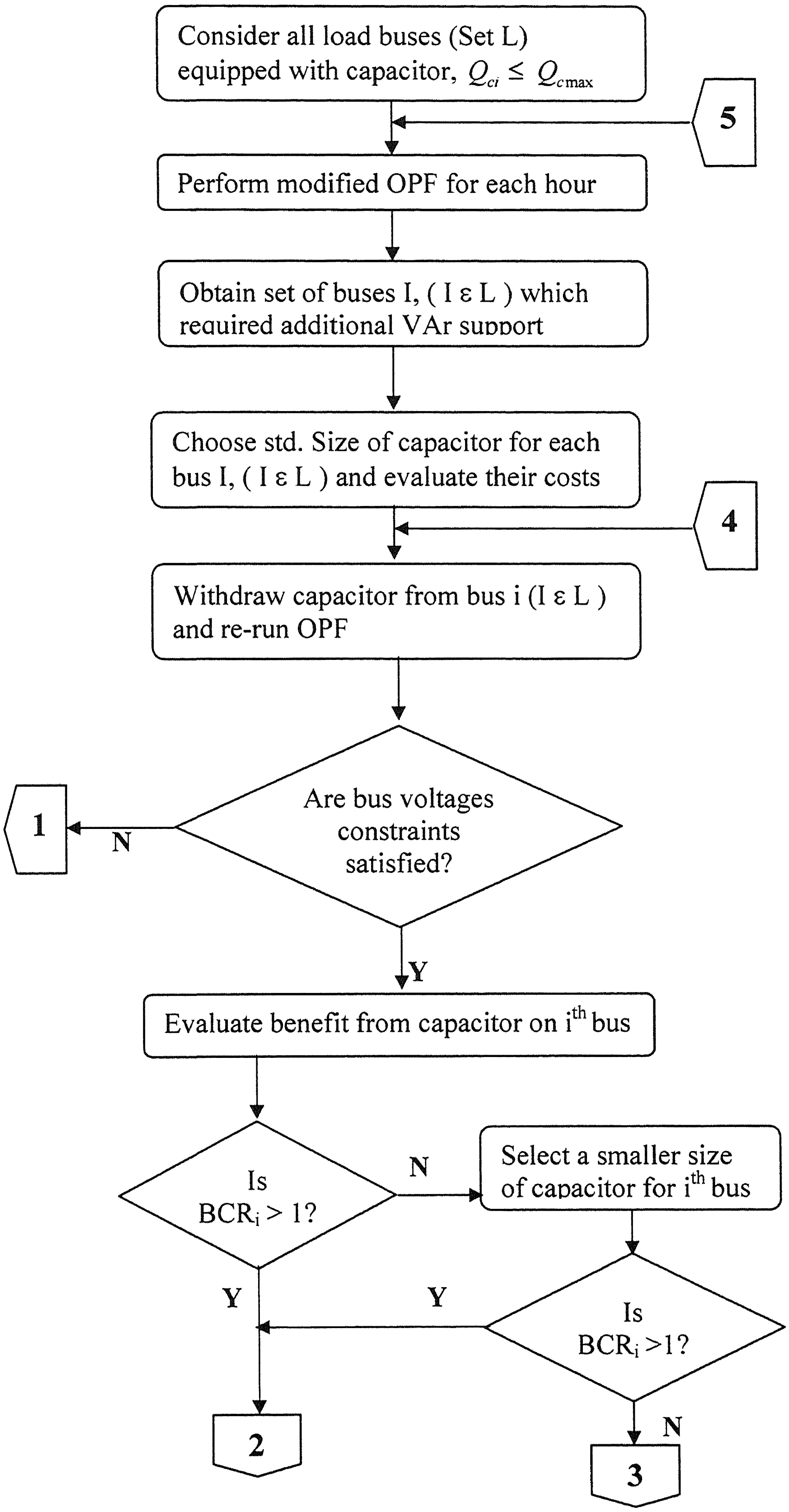
$\nu_{i,max}$: Lagrange multiplier on the max. voltage level at i^{th} bus

$\sigma_{i,min}$: Lagrange multiplier on the min. reactive power generation limit from capacitor bank at i^{th} bus

$\sigma_{i,max}$: Lagrange multiplier on the max reactive power generation limit from capacitor bank at i^{th} bus

The OPF solution is obtained by applying Kuhn-Tucker conditions for the minimization problem.

2.3 Flow chart



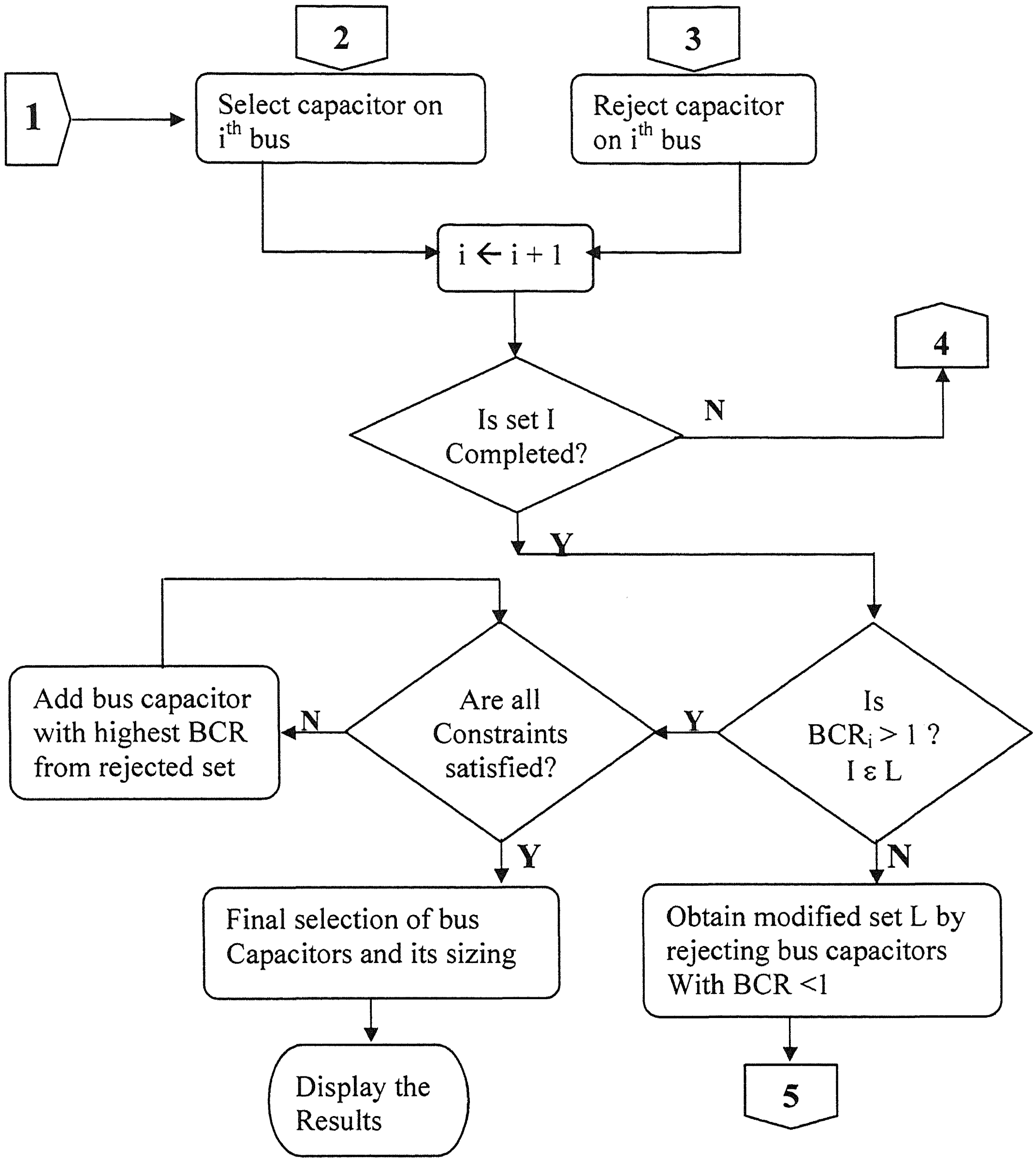


Fig.2.1: Flow chart for optimal allocation and sizing of capacitors

2.4 System Studies and Discussions

The studies were conducted on following two systems for the application of methodology [18]:

(i) IEEE-14 bus system as described in Appendix-A

(ii) IEEE-118 bus system as described in Appendix-B.

2.4.1 IEEE-14 Bus system

The IEEE-14 Bus system has been analyzed in details considering detailed hourly loading conditions at each bus. The optimal allocation, sizing of capacitors and CBA scheme with the results obtained from the formulated model is described below.

Step 1: The modified OPF is solved for each hourly loading condition considering the objective function (2.1). All load buses are initially equipped with capacitors with Q_c^{max}= 2.0 p.u. MVAr. It gives the amount of reactive power support required at each load buses at different loading conditions over a daily load curve. The results obtained from the formulated model are shown in Table 2.1 (bus #4 and bus #7 which are load buses not shown here because the capacitor values at these buses are zeros in all load conditions):

Table 2.1: IEEE-14 bus system- VAr requirements at load buses for different LSF

LSF*	Capacitors values (p.u. MVAr) on base MVA=100						
	BUS#5	BUS#9	BUS#10	BUS#11	BUS#12	BUS#13	BUS#14
0.7	0.1144	-----	0.0025	-----	-----	0.0114	0.0282
0.8	0.1284	-----	0.0178	0.0036	0.0054	0.0328	0.0349
0.9	0.1367	-----	0.0280	0.0082	0.0091	0.0430	0.0415
1.0	0.1417	-----	0.0430	0.0102	0.0108	0.0496	0.0495
1.1	0.1444	-----	0.0581	0.0121	0.0125	0.0561	0.0577
1.2	0.1479	0.0005	0.0708	0.0142	0.0144	0.0631	0.0651
1.3	0.1473	0.0214	0.0766	0.0162	0.0165	0.0699	0.0710
1.4	**0.1472**	**0.0437**	**0.0825**	**0.0182**	**0.0186**	**0.0769**	**0.0768**

* Load scale factor

Step 2: Table 2.1 shows the VAr requirements at each load buses for different loading conditions over a daily load curve. Hence the preliminary selection of capacitor at a load buses is made on the basis of the maximum VAr requirements at that buses over a daily load curve. Then calculate the capital cost of capacitor units per day considering the standard unit of capacitor banks. The calculation of cost of capacitor unit is described in Appendix C.

The cost ≈ \$1.4136/MVAr/Day

Given below are the preliminary selection of capacitors at load buses and their capital cost:

Table 2.2: IEEE-14 bus system- Capacitors at load buses and their capital cost on per day basis

Load Bus	Qc(MVAr)	Cost/Day
5	15	21.2040
9	4	5.65440
10	8	11.3088
11	2	2.82720
12	2	2.82720
13	8	11.3088
14	8	11.3088

Step 3: As seen from Table 2.1 from step-1, the reactive power support required at load buses increases as loading on the system increases. However, it is worth to examining whether installation of capacitor banks at these buses would be really cost-effective or not. To this effect, a cost benefit analysis (*CBA*) is carried out. So the marginal benefit from capacitor addition on i^{th} bus is calculated by running modified OPF with capacitor units at all load buses as carried out in Step-1 and without capacitor on i^{th} bus ($i \in L$). The difference in system cost is the marginal benefit from capacitor addition on i^{th} bus. The results from the model are given in Table 2.3:

Table 2.3: IEEE-14 bus system- Marginal benefits in \$

LSF	BUS#5	BUS#9	BUS#10	BUS#11	BUS#12	BUS#13	BUS#14
0.7	0.43	0	0	0	0	0.04	0.20
0.8	0.61	0	0.02	0	0.01	0.18	0.32
0.9	0.77	0	0.09	0.01	0.02	0.27	0.49
1.0	0.83	0	0.21	0.01	0.03	0.37	0.72
1.1	0.88	0	0.40	0.03	0.05	0.48	0.99
1.2	0.88	0	0.43	0.04	0.07	0.61	1.20
1.3	0.87	0.02	0.50	0.04	0.08	0.75	1.44
1.4	0.87	0.08	0.59	0.06	0.11	0.92	1.71
Total Benefit for the day (\$)	**18.42**	**0.3**	**6.72**	**0.57**	**1.11**	**10.86**	**21.21**

Step 4: The benefit to cost ratio (BCR) for the load buses are described in below table:

Table 2.4: IEEE-14 bus system- BCR for the load buses

Load Bus	Qc(MVAr)	Cost/Day	Benefit	BCR
5	15	21.204	18.42	***0.8687***
9	4	5.6544	0.3	0.0531
10	8	11.3088	6.72	0.5942
11	2	2.8272	0.57	0.2016
12	2	2.8272	1.11	0.3926
13	8	11.3088	10.86	***0.9603***
14	8	11.3088	21.21	**1.8755**

Since the Benefit-to-Cost ratio for a capacitor on bus #14 is more than one, thus bus #14 is selected for installation of capacitor.

Step 5: Obtain the modified set L to be considered for installation of capacitors by rejecting load buses whose BCR value less than one.

Step 6: An OPF computation is carried out considering capacitors on load buses excluding load buses whose BCR value less than one. It is observed that, the voltage constraints are violated for higher loading. Therefore, additional reactive power support is needed at some other load buses, inspite of their low BCR.

Step 7: From Step-4, it is seen that the capacitor on bus #13 has higher BCR as compared to that on other buses. Hence bus #13 is selected for installing capacitor. Again OPF is performed and observed that again the voltage constraints are violated for higher loading. Hence next highest BCR of load bus #5 is considered for installing capacitor. Another OPF computation ensures that this selection gives feasible solution. Thus this solution itself is the final optimal solution. The final result obtained from the model is shown in Table 2.5.

Table 2.5: IEEE-14 bus system- With optimal size of capacitors at load buses #14, #13 and #5 for different LSF

LSF	Capacitors values (p.u.MVAr) on base MVA=100		
	BUS#5	BUS#13	BUS#14
0.7	0.11412	0.011795	0.028191

0.8	0.12808	0.038288	0.035459
0.9	0.14586	0.046034	0.045321
1.0	0.1555	0.053245	0.055189
1.1	0.16295	0.060409	0.065237
1.2	0.17616	0.067987	0.073559
1.3	0.19432	0.07567	0.081597
1.4	0.21342	0.083488	0.089965

2.4.2 IEEE-118 Bus system

The IEEE-118 bus system is considered to demonstrate this phenomenon for one load condition (peak load, LSF =1.4). The reactive power demands are increased considering a power factor of 0.85 at all buses to represent a heavily reactive power loaded system. The same scheme as described above is also applied here in order to achieve a final optimal solution.

The IEEE-118 bus system has 64 load buses, initially all load buses are considered for capacitor installation. The below Table 2.6 describes the results of first OPF computation with benefit to cost ratio analysis on IEEE-118 bus system.

Table 2.6: Selection of bus capacitor for IEEE-118 bus system (1^{st} iteration)

Initial choice of capacitor at load bus		Buses with BCR>1		Buses rejected from {L} i.e., BCR<1	
Load Bus	Cap. (MVAr)	Load Bus	Cap. (MVAr)	Load Bus	Cap (MVAr)
2	10	2	10	20	3
3	19	3	19	22	9
11	47	11	47	30	16
13	35	13	35	78	37
14	16	14	16		
16	15	16	15		
20	3	21	11		
21	11	28	12		
22	9	29	15		
28	12	33	28		
29	15	38	3		
30	16	39	14		

33	28	41	25	
38	3	43	12	
39	14	44	1	
41	25	45	37	
43	12	47	25	
44	1	50	12	
45	37	51	16	
47	25	52	16	
50	12	53	17	
51	16	57	18	
52	16	58	18	
53	17	60	119	
57	18	63	89	
58	18	67	67	
60	119	71	4	
63	89	75	37	
67	67	79	11	
71	4	82	19	
75	37	83	1	
78	37	84	6	
79	11	86	5	
82	19	88	40	
83	1	93	11	
84	6	94	24	
86	5	95	37	
88	40	96	15	
93	11	97	8	
94	24	98	22	
95	37	101	14	
96	15	102	6	
97	8	106	14	
98	22	114	2	
101	14	115	15	
102	6	117	15	
106	14	118	24	
114	2			
115	15			
117	15			
118	24			

As seen from Table 2.6, after the first OPF computations out of 64 load buses only 51 buses were selected for VAr support. However, only capacitors on buses #20, #22, #30 and #78 had a BCR<1 and hence these buses were rejected from the set of candidate

buses {L}. Again OPF computations were carried with set of candidate buses excluding the buses whose BCR value less than one. The results from the OPF computation are described below:

Table 2.7: Selection of bus capacitor for IEEE-118 bus system (2^{nd} iteration)

Initial choice of cap. at load bus		Buses with BCR>1		Buses rejected from {L} i.e., BCR<1	
Load Bus	Cap. (MVAr)	Load Bus	Cap. (MVAr)	Load Bus	Cap (MVAr)
2	10	3	18	2	10
3	18	11	47	115	15
11	47	13	36		
13	36	14	17		
14	17	16	15		
16	15	21	20		
21	20	28	12		
28	12	29	15		
29	15	33	30		
33	30	38	6		
38	6	39	14		
39	14	41	25		
41	25	43	12		
43	12	44	1		
44	1	45	37		
45	37	47	25		
47	25	50	12		
50	12	51	16		
51	16	52	16		
52	16	53	17		
53	17	57	18		
57	18	58	18		
58	18	60	119		
60	119	63	89		
63	89	67	67		
67	67	71	4		
71	4	75	37		
75	37	79	26		
79	26	82	19		
82	19	83	1		
83	1	84	6		
84	6	86	5		

86	5	88	40	
88	40	93	11	
93	11	94	24	
94	24	95	37	
95	37	96	15	
96	15	97	8	
97	8	98	25	
98	25	101	14	
101	14	102	6	
102	6	106	14	
106	14	114	2	
114	2	117	15	
115	15	118	24	
117	15			
118	24			

In second OPF computations, again two buses (#2 & #115) were rejected from the set of candidate buses {L}. So the iteration process is continued till the optimal solution is achieved. The results from the model with BCR analysis are shown in subsequent table.

Table 2.8: Selection of bus capacitor for IEEE-118 bus system (3rd iteration)

Initial choice of cap. at load bus		Buses with BCR>1		Buses rejected from {L} i.e., BCR<1	
Load Bus	Cap. (MVAr)	Load Bus	Cap. (MVAr)	Load Bus	Cap (MVAr)
3	19	3	19	28	12
11	52	11	52		
13	35	13	35		
14	18	14	18		
16	17	16	17		
21	20	21	20		
28	12	29	15		
29	15	33	29		
33	29	38	6		
38	6	39	14		
39	14	41	25		
41	25	43	12		
43	12	44	1		
44	1	45	37		
45	37	47	25		
47	25	50	12		

50	12	51	16	
51	16	52	16	
52	16	53	17	
53	17	57	18	
57	18	58	18	
58	18	60	119	
60	119	63	89	
63	89	67	67	
67	67	71	4	
71	4	75	37	
75	37	79	26	
79	26	82	19	
82	19	83	1	
83	1	84	6	
84	6	86	5	
86	5	88	40	
88	40	93	11	
93	11	94	24	
94	24	95	37	
95	37	96	15	
96	15	97	8	
97	8	98	23	
98	23	101	14	
101	14	102	6	
102	6	106	14	
106	14	114	14	
114	14	117	16	
117	16	118	24	
118	24			

Table 2.9: Selection of bus capacitor for IEEE-118 bus system (4th iteration)

Initial choice of cap. at load bus		Buses with BCR>1		Buses rejected from {L} i.e., BCR<1	
Load Bus	Cap. (MVAr)	Load Bus	Cap. (MVAr)	Load Bus	Cap (MVAr)
3	19	3	**19**	**None**	
11	52	11	**52**		
13	35	13	**35**		
14	18	14	**18**		
16	17	16	**17**		
21	20	21	**20**		
29	20	29	**20**		

33	29	33	**29**	
38	6	38	**6**	
39	14	39	**14**	
41	25	41	**25**	
43	12	43	**12**	
44	1	44	**1**	
45	37	45	**37**	
47	25	47	**25**	
50	12	50	**12**	
51	16	51	**16**	
52	16	52	**16**	
53	17	53	**17**	
57	18	57	**18**	
58	18	58	**18**	
60	119	60	**119**	
63	89	63	**89**	
67	67	67	**67**	
71	4	71	**4**	
75	37	75	**37**	
79	26	79	**26**	
82	19	82	**19**	
83	1	83	**1**	
84	6	84	**6**	
86	5	86	**5**	
88	40	88	**40**	
93	11	93	**11**	
94	24	94	**24**	
95	37	95	**37**	
96	15	96	**15**	
97	8	97	**8**	
98	23	98	**23**	
101	14	101	**14**	
102	6	102	**6**	
106	14	106	**14**	
114	14	114	**14**	
117	16	117	**16**	
118	24	118	**24**	

We observed that as iteration process continues, the number of buses in last column start decreasing and the iterative scheme converges to the final solution when all the buses selected for capacitor placement have BCR more than 1. It is to be noted that, only a subset of buses are considered for selection which gradually reduces, till the final solution

is attained, when no further bus capacitors are rejected. Therefore, from BCR analysis, finally 44 load buses were selected for installation of capacitor units.

2.5 Conclusion

In this chapter, a simple approach to reactive power planning has been presented in details. The methodology has been implemented using a modified optimal power flow. The criterion used in the reactive power planning is to minimize the system generating cost and the cost of adding new capacitors. All load buses have been initially considered as possible locations for installation of capacitors. The optimal placement of capacitors and their sizes at the load buses is decided from the cost benefit analysis (CBA). Due to which the utilities can reduce their capital investment on reactive power sources and same time ensuring the reliable operation of the power system and also get the maximum benefit from the limited reactive power sources. Based on the results obtained on the IEEE-14 bus system and IEEE-118 bus systems, the following main conclusions can be drawn.

- This is simple approach to reactive power planning
- As we have seen in IEEE-118 bus system compare to IEEE-14 bus system more number of capacitor unit were selected for supporting the system because of heavy reactive loading.
- As we have seen in IEEE-118 bus system compare to IEEE-14 bus system, the rejected buses were less in number because of the basic property of reactive power is highly location dependent and moreover, we considered a heavy reactive power loading on the system. Due to which, more number of capacitor units are required to support the system so that the system can operate reliably.

Chapter 3

Reactive Power Pricing Analysis

3.1 Introduction

During last two decades electric power systems around the world have been continuously evolving and experiencing important changes because of privatization and deregulation process. Thus planning and operation of the utilities are based on the economic principles of open-access markets. In this new environment, electric markets are essentially competitive. A fair and adequate method for allocating the costs may help the market participants make appropriate and efficient investments of reactive power sources, which can offer system operators more tools and can strengthen the system security [24].

According to the Federal Energy Regulatory Commission (FERC), a fixed tariff on the remuneration for reactive power is insufficient to provide a proper signal of reactive power cost [19]. Berg. et al. [20] pointed out the inconsistency and inadequacy of the pricing policies based on power factor penalties. They suggest that given the present of high cost of additional investments by electric utilities, price should be derived from economic principles, which support a pricing approach that has price equal marginal costs that would also reflect today's technological constraints. It is also realized that establishing an accurate pricing structure of reactive power can not only recover the costs of reactive power providers, but also provide economic information for real-time operations.

The real time pricing method of active power was established by Schweppe et al. [3]. They suggested spot pricing can help to improve production efficiency and yield maximum social benefits. In [22] spot pricing becoming more attractive in a competitive electricity market, especially at the generation stage where independent power producers would adjust their generation levels according to the spot prices paid to the producers for their power production.

As power system margins are reduced because of emphasis on the greater use of generation and transmission, power system dispatchers must operate their system much closer to their technical limits. Reactive power pricing in real time (spot-pricing) addresses the important issue of providing information to both the utility and consumers about the true burden on the system, in terms of cost and other system parameters viz. voltage drops and increases transmission losses, from time to time. In [8] real-time pricing of reactive power has been shown to perform better than the power factor penalty scheme in terms of providing incentives to all customers to reduce their consumption of reactive power irrespective of their power factor which are extended from the active power marginal pricing structure in [3].

A number of applications were developed for calculating reactive power marginal prices based on marginal theory. The results indicate that the marginal theory gives signals to consumers to reduce their reactive power demands. Consequently reactive power marginal price are local signals that vary depending on the bus location. However, these prices represent a small portion of the actual reactive power price [8, 15, 23, and 29].

A comprehensive extension of spot pricing is discussed in [26-27]. Chattopadhyay et al. [15] pointed out that reactive power price should recover not only the operational cost, but also capital investment of capacitors. Hao and Papalexopoulous make a detailed discussion on reactive power services and argue that reactive power marginal price is typically less than 1 percent of the active power marginal price in a well designed system and depends strongly on the network constraints. Therefore, the capital costs incurred as part of the reactive power service should be used in the reactive power price calculation

[21]. Hence capital investment of capacitors also included along with system operating cost for calculating reactive power prices.

The mathematical model used to calculate active and reactive power marginal prices by a modified OPF formulation is already discussed in section 2.2 of chapter 2.

In this chapter, the impact of various factors on both active and reactive power marginal prices i.e., change of objective function and different system operating conditions including load power factor, daily load fluctuation and voltage control have been studied in details to observe how these conditions influence both active and reactive power prices. Case studies have been conducted on IEEE-14 and IEEE-118 bus systems.

3.2 The Price of Electricity

According to microeconomics, the marginal prices for active and reactive powers at bus i are the marginal costs associated with the corresponding load flow equations when the OPF is solved as a non-linear programming problem. At a particular time, real price of real power and that of reactive power, at a bus-i are given by,

$$\rho_i^p = \frac{\partial L}{\partial P_i} = MC_{pi} \tag{3.1}$$

$$\rho_i^q = \frac{\partial L}{\partial Q_i} = MC_{qi} \tag{3.2}$$

Where,

P_i, Q_i: Net injected real and reactive powers at bus-i.

MC_{pi}, MC_{qi}: Marginal costs are equal to the Lagragian multipliers of the corresponding power flow equations (2.3) and (2.4 & 2.5) at the optimal solution point.

3.3 Case studies and Discussions

The studies have been conducted on the IEEE-14 and IEEE-118 bus systems. The details of these systems are given in Appendix-A and B respectively. The detailed results of the studies conducted on these two systems are discussed in following sections.

The capacitor units' installation at the load buses are taken from the results obtained in section 2.4 of Chapter 2. For present studies, we assume that reactive power output of capacitor units' can be adjusted continuously and analyses were carried on base case operation of the systems.

In order to study the impacts of various factors on the marginal price of reactive power and also active power (in some cases), five cases are studied:

1. The objective function has all the two items as described in (2.1)
2. The objective function has only the first item of (2.1)

Based on case 1, again three cases are designed to study the impacts of various factors on reactive power marginal prices. Those cases are:

3. Load power factor.
4. Daily load fluctuation (or change in system operating point).
5. Voltage control.

3.3.1 IEEE-14 Bus System

The IEEE-14 bus system has been taken from the reference [18]. From Chapter 2, we concluded that out of 9 load buses only 3 load buses (#5, #13 and #14) of IEEE-14 bus system were selected for suitable placement for capacitor units. Therefore, capacitor units are placed at those buses for present analysis.

3.3.1.1 Impact of change of objective function (case 1 and 2)

The results obtained from OPF model for case 1 and case 2 are given in Table 3.1, where ρ_{q_avg}, the average cost of reactive power of the whole network and is obtained through dividing the total system reactive power cost by the total reactive power demand.

Table 3.1: Comparison of results of IEEE-14 bus system for case 1 & case 2

	Case 1	Case 2
Objective function	$COST = \sum_{i\in NG} C_i(P_{gi}) + \sum_{i\in NL} Q_{ci}.CAPCOST$	$COST = \sum_{i\in NG} C_i(P_{gi})$
Total active power production cost of generators	\$8076.5261/hr	\$8076.0243/hr
Total capital cost of capacitors	\$1.55456/hr	-----
ρ_{q_avg}	\$0.0211505/MVAr-Hr	-----

Bus No.	Generators output (MVA)				Output of Capacitor units (MVAr)		Active & reactive power marginal prices			
	Case 1		Case 2		Case 1	Case 2	Case 1		Case 2	
	Real power	Reactive power	Real power	Reactive power	Reactive power	Reactive power	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)
1	194.42	0	194.45	0			36.731	0.164	36.734	0.194
2	36.73	12.37	36.74	7.82			38.367	0	38.37	0
3	28.83	22.39	28.88	21.8			40.577	0	40.578	0
6	0	-0.02	0	-3.3			39.722	0	39.719	0
8	8.19	6.02	8.09	5.6			40.164	0	40.162	0
4							40.191	0.016	40.192	0.019
7							40.164	0.038	40.162	0.014
9							40.153	0.075	40.148	0.042
10							40.301	0.203	40.297	0.174
11							40.141	0.17	40.138	0.155
12							40.36	0.125	40.358	0.107
5					15.5498	22.7197	39.664	0.059	39.666	0
13					5.3245	6.7498	40.55	0.059	40.546	0
14					5.5189	5.7898	41.145	0.059	41.137	0

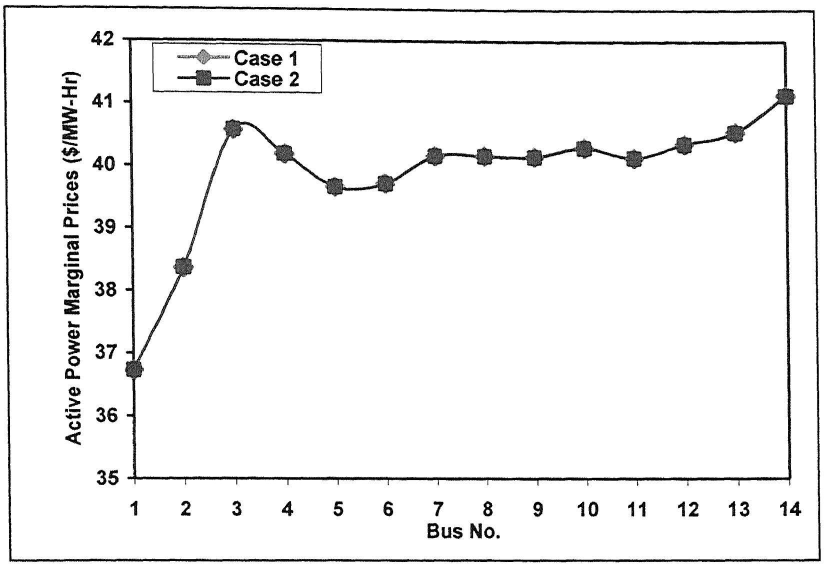


Fig. 3.1: Comparison of active power marginal prices of IEEE-14 bus system for case 1 & case 2

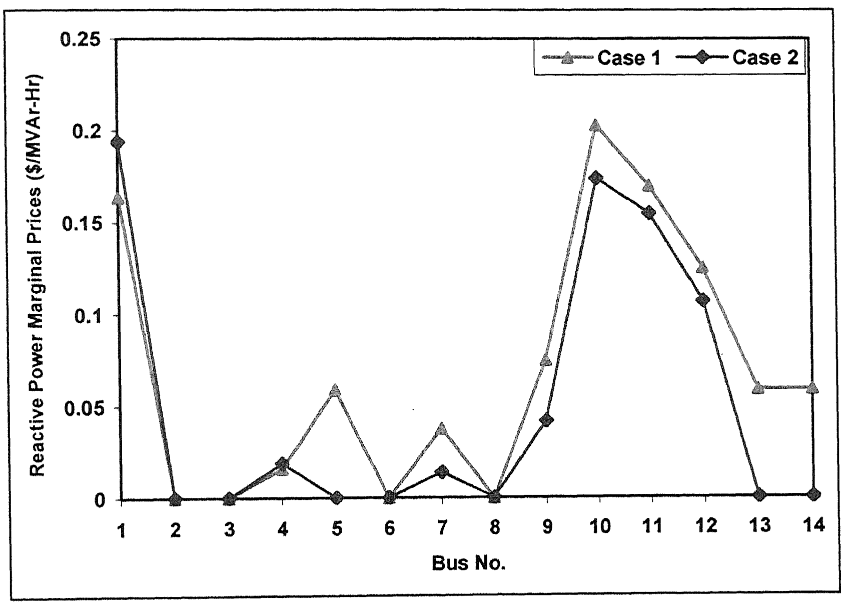


Fig. 3.2: Comparison of reactive power marginal prices of IEEE-14 bus system for case 1 & case 2

From Table 3.1 and Fig. 3.1 & 3.2, we observed the following facts:

1. The total active power production cost & active power marginal prices at various buses have only a small change when objective function changes. Therefore,

active power pricing sub-problem can be studied independently with reactive production cost neglected.

2. For each case, active power marginal prices at various buses are in the same order while reactive power marginal prices (RPMP) fluctuate significantly from bus to bus. Generally active power marginal price is much higher than RPMP at a certain buses.
3. When the capacitor cost is neglected, the corresponding reactive power source bus (es) will have zero or very little RPMP(s) for the free reactive power available locally. The nearby buses also get benefited and have small RPMPs.
4. When reactive power production cost is taken into consideration, the corresponding RPMP increase noticeably.
5. The revenue from RPMP will be higher than that from the system average price of reactive power. Therefore, some adjustment should be made accordingly if RPMP is to be used.

3.3.1.2 Impact of load power factor (Case 3)

In this case, the power factor of the load is varied from 0.7 to 0.95 i.e., from high reactive power loading to near to upf loading and it's impact on RPMPs, voltage profiles and reactive power output of generators and capacitors have been studied and results are described in below tables and figures.

Table 3.2: Load pf- reactive power marginal prices of IEEE-14 bus system

	Reactive power marginal prices ($/MVAr-Hr)					
Bus No.	Pf=0.7	Pf=0.75	Pf=0.8	Pf=0.85	Pf=0.9	Pf=0.95
1	0	0.137	0.172	0.167	0.157	0.152
2	0.257	0.056	0	0	0	0
3	1.893	1.303	0.882	0.538	0.207	0
4	0.323	0.204	0.131	0.077	0.025	0.003
5	0.059	0.059	0.059	0.059	0.059	0.059
6	0	0	0	0	0	0.001
7	0.229	0.16	0.117	0.08	0.026	0
8	0	0	0	0	0	0
9	0.328	0.24	0.186	0.133	0.045	0.001
10	0.52	0.41	0.331	0.253	0.145	0.071

11	0.403	0.327	0.268	0.21	0.135	0.076
12	0.431	0.372	0.315	0.262	0.212	0.154
13	0.059	0.059	0.059	0.059	0.059	0.059
14	0.059	0.059	0.059	0.059	0.059	0.059
Avg. cost	0.02898	0.02384	0.02123	0.019912	0.019242	0.019

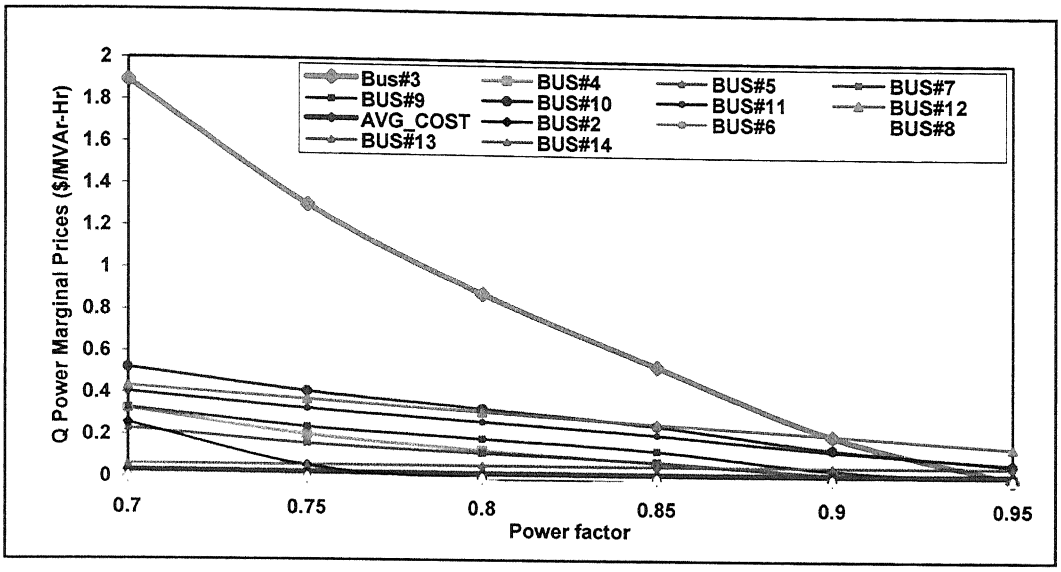


Fig. 3.3: Load pf-reactive power marginal prices and the average cost of IEEE-14 bus system

Table 3.3: Load pf- voltage magnitude profiles of IEEE-14 bus system

Bus No.	Voltage magnitudes (p.u.)					
	Pf = 0.7	Pf = 0.75	Pf= 0.8	Pf = 0.85	Pf = 0.9	Pf=0.95
1	1.06	1.06	1.06	1.06	1.06	1.06
2	1.038	1.039	1.04	1.04	1.039	1.039
3	0.954	0.967	0.98	0.993	1.007	1.015
4	1.01	1.009	1.011	1.014	1.017	1.018
5	1.028	1.022	1.021	1.021	1.022	1.022
6	1.045	1.048	1.054	1.06	1.06	1.06
7	1.036	1.039	1.043	1.048	1.051	1.05
8	1.06	1.06	1.06	1.06	1.06	1.049
9	1.025	1.031	1.038	1.047	1.052	1.056
10	1.018	1.024	1.033	1.041	1.047	1.051
11	1.026	1.031	1.039	1.047	1.05	1.053
12	1.028	1.032	1.039	1.046	1.047	1.049
13	1.031	1.034	1.041	1.047	1.047	1.048
14	1.021	1.025	1.031	1.038	1.04	1.041

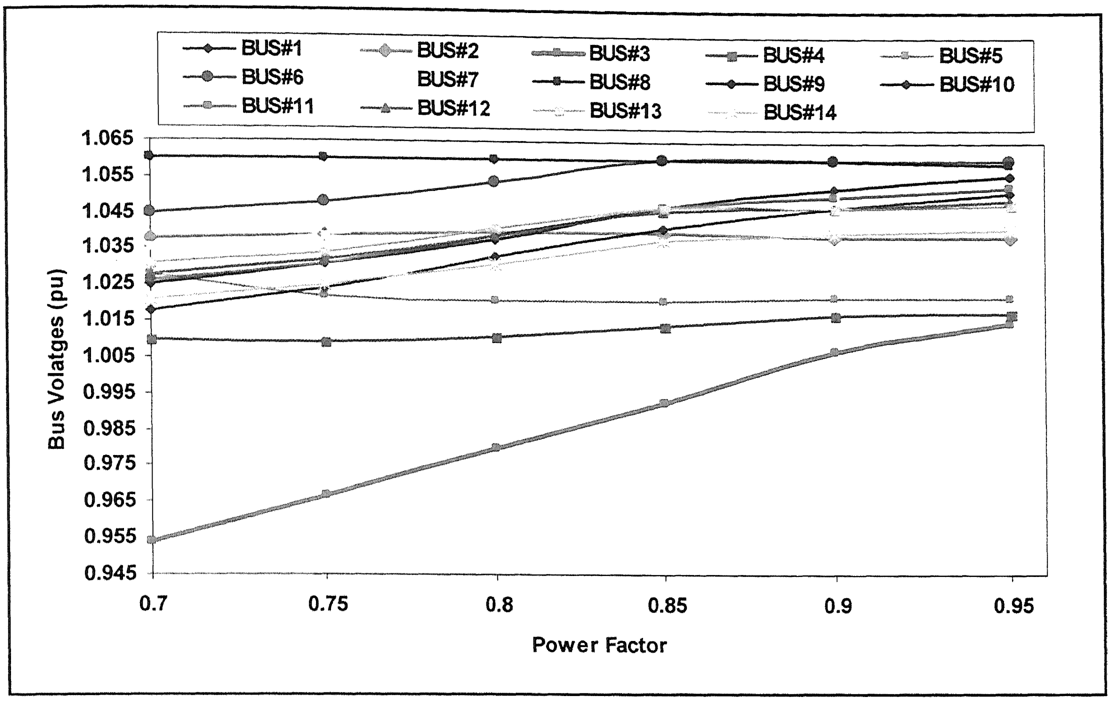


Fig. 3.4: Load pf- voltage magnitude profiles of IEEE-14 bus system

Table 3.4: Load pf- reactive power output of generators and capacitors of IEEE-14 bus system

Gen. bus No.	Generator reactive power output (MVAr)					
	Pf = 0.7	Pf = 0.75	Pf = 0.8	Pf = 0.85	Pf = 0.9	Pf = 0.95
1	0.35	0	0	0	0	0
2	50	50	40.81	28.46	16.41	7.39
3	40	40	40	40	40	34.03
6	2.71	3.31	3.46	2.39	-2.03	-6
8	14.25	12.76	10.22	7.51	5.47	-0.35
Cap. bus	Capacitor reactive power output (MVAr)					
5	70.9649	45.7672	32.1986	23.9747	18.281	15.1829
13	14.8356	12.684	10.6245	8.5858	6.4559	4.0174
14	18.2821	15.3545	12.8484	10.368	7.4016	4.5917

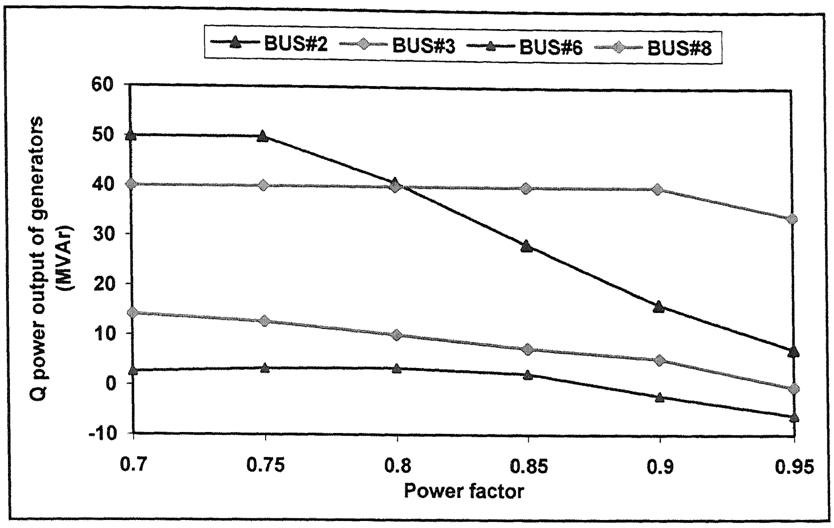


Fig. 3.5: Load pf- reactive power output of generators of IEEE-14 bus system

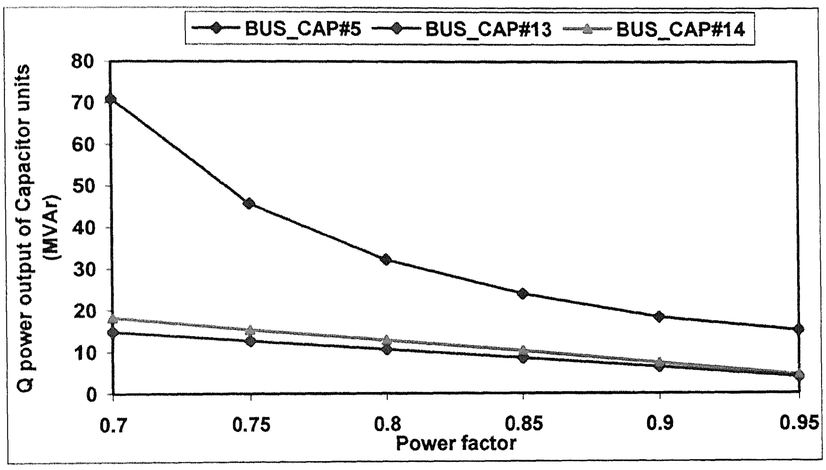


Fig. 3.6: Load pf- reactive power output of capacitor units of IEEE-14 bus sytem

Base on above Tables and Figures of case 3, the following facts are observed:

1. When load pf reduces from 0.95 to 0.7, the RPMP increases greatly while average price increases very slow. Therefore, RPMP can provide clear economic information to loads to improve their power factors.
2. When bus #3 reaches its minimum voltage of 0.95pu at lower pf, the corresponding RPMP of bus #3 increase drastically which can act as an index of the urgency of the reactive power supply and voltage support on bus #3.

3. When the pf is close to upf, the Q power output of bus 6 and 8 become negative. This means that the system has surplus reactive power. The corresponding RPMP is very small.
4. When the cheaper and local Q source of capacitor is used up, the load bus voltages will reduce quickly along with the pf reduction and the corresponding RPMP will increases rapidly at the same time.
5. The revenue of reactive power supply based on marginal price will be much more than that based on average price especially at lower pf. Therefore, some adjustment should be made if RPMP is going to be used.

3.3.1.3 Impact of daily load fluctuations (Case 4)

In this case, impact of change in system operating point on RPMPs and also on active power marginal prices have been studied by considering the daily load percentage change is in a pattern as shown in Fig. 3.7 and all the load power factor keeps as 0.9. The results are given in below tables and figures.

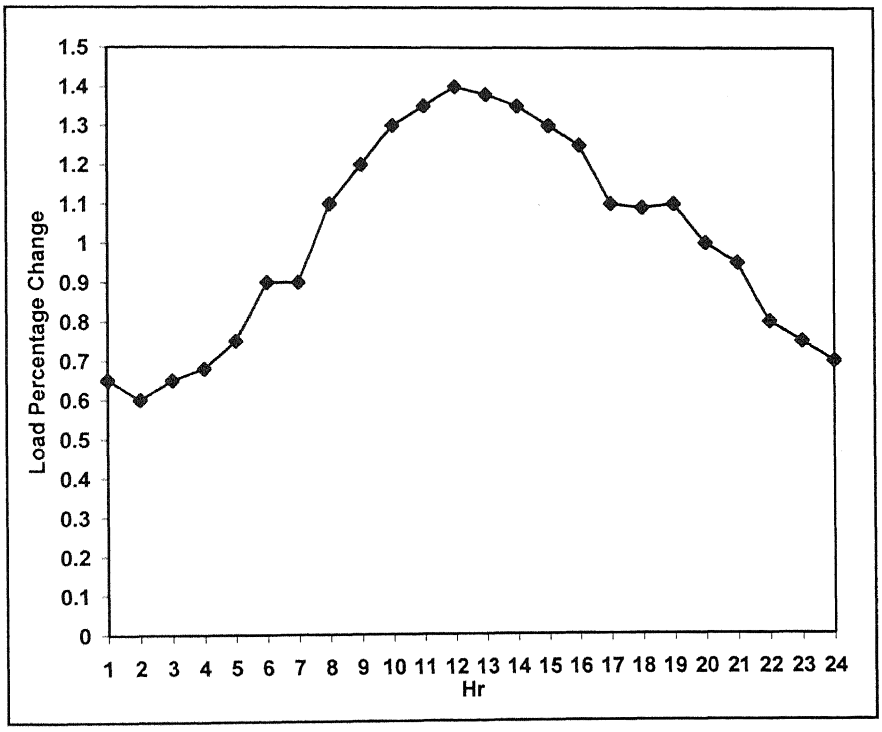


Fig. 3.7: Daily load change

Table 3.5: Daily load fluctuation - active power marginal prices of IEEE-14 bus system

Hrs	Active power marginal prices ($/MW-hr)													
	Generator Buses					Load Buses								
	1	2	3	6	8	4	5	7	9	10	11	12	13	14
1	32.599	33.681	35.324	34.433	34.809	34.806	34.445	34.809	34.808	34.87	34.728	34.793	34.915	35.309
2	31.609	32.564	34.019	33.206	33.56	33.554	33.232	33.56	33.561	33.612	33.478	33.524	33.636	33.997
3	32.599	33.681	35.324	34.433	34.809	34.806	34.445	34.809	34.808	34.87	34.728	34.793	34.915	35.309
4	33.195	34.356	36.12	35.178	35.568	35.568	35.182	35.568	35.568	35.636	35.489	35.564	35.693	36.108
5	34.595	35.944	38.002	36.916	37.357	37.359	36.911	37.357	37.355	37.436	37.27	37.363	37.511	37.987
6	36.422	38.018	40.304	39.262	39.778	39.783	39.248	39.778	39.776	39.887	39.694	39.834	40.018	40.605
7	36.422	38.018	40.304	39.262	39.778	39.783	39.248	39.778	39.776	39.887	39.694	39.834	40.018	40.605
8	36.923	38.583	40.806	40.019	40.384	40.436	39.917	40.386	40.363	40.547	40.422	40.726	40.918	41.513
9	37.053	38.739	40.991	40.148	40.556	40.599	40.064	40.559	40.545	40.746	40.601	40.925	41.14	41.808
10	37.182	38.895	41.176	40.278	40.727	40.763	40.212	40.733	40.728	40.946	40.781	41.125	41.364	42.107
11	37.247	38.973	41.27	40.342	40.813	40.845	40.287	40.82	40.819	41.047	40.871	41.225	41.477	42.258
12	37.312	39.051	41.363	40.407	40.898	40.927	40.361	40.907	40.912	41.149	40.961	41.326	41.59	42.41
13	37.286	39.019	41.326	40.381	40.864	40.894	40.331	40.872	40.875	41.108	40.925	41.285	41.545	42.349
14	37.247	38.973	41.27	40.342	40.813	40.845	40.287	40.82	40.819	41.047	40.871	41.225	41.477	42.258
15	37.182	38.895	41.176	40.278	40.727	40.763	40.212	40.733	40.728	40.946	40.781	41.125	41.364	42.107
16	37.117	38.817	41.083	40.213	40.642	40.681	40.138	40.646	40.636	40.846	40.691	41.024	41.252	41.957
17	36.923	38.583	40.806	40.019	40.384	40.436	39.917	40.386	40.363	40.547	40.422	40.726	40.918	41.513
18	36.91	38.568	40.787	40.007	40.367	40.42	39.902	40.368	40.345	40.527	40.404	40.706	40.896	41.484
19	36.923	38.583	40.806	40.019	40.384	40.436	39.917	40.386	40.363	40.547	40.422	40.726	40.918	41.513
20	36.721	38.353	40.584	39.708	40.159	40.185	39.654	40.16	40.148	40.294	40.131	40.348	40.538	41.134
21	36.615	38.232	40.47	39.537	40.042	40.052	39.513	40.042	40.038	40.164	39.976	40.145	40.335	40.939
22	35.601	37.092	39.377	38.178	38.658	38.662	38.166	38.658	38.655	38.746	38.566	38.672	38.835	39.356
23	34.595	35.944	38.002	36.916	37.357	37.359	36.911	37.357	37.355	37.436	37.27	37.363	37.511	37.987
24	33.594	34.808	36.653	35.672	36.076	36.076	35.673	36.076	36.075	36.147	35.995	36.075	36.209	36.641

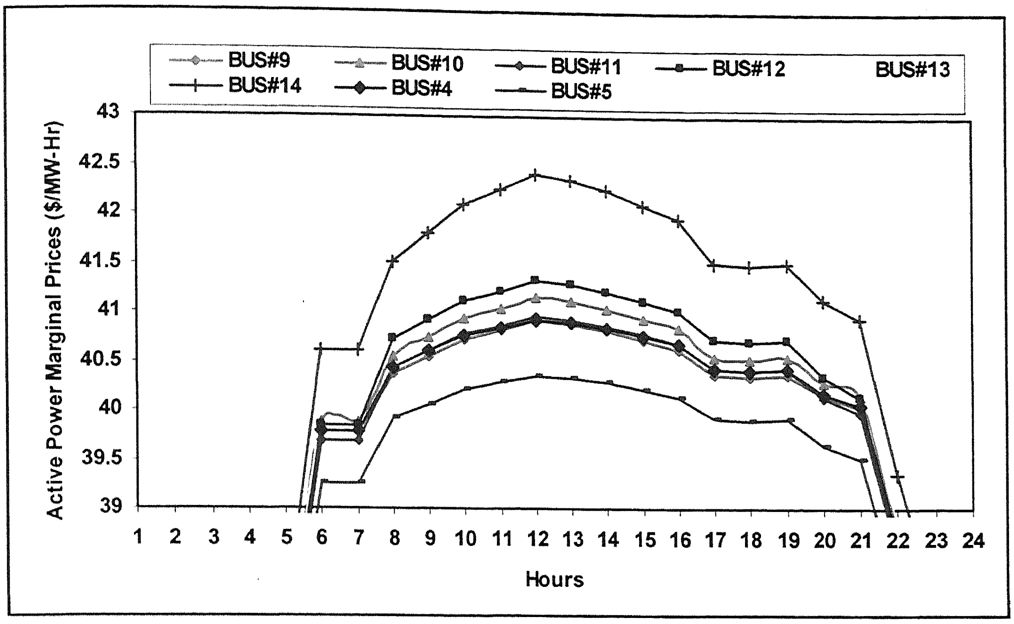


Fig. 3.8: Daily load fluctuation - active power marginal prices of IEEE-14 bus system

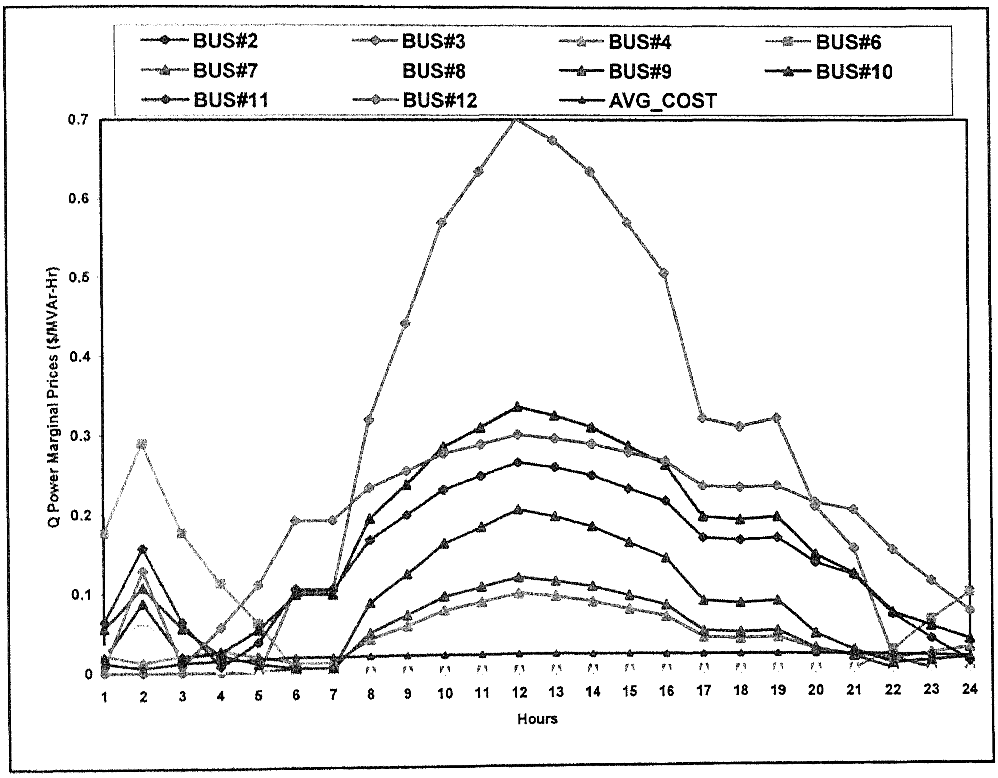


Fig. 3.9: Daily load fluctuation-reactive power marginal prices of IEEE-14 bus system

Table 3.6: Daily load fluctuation - reactive power marginal prices of IEEE-14 bus system

Hrs	Reactive power marginal prices ($/MVAr-hr)														Avg. cost ($/MVAr-hr)
	Generator Buses					Load Buses									
	1	2	3	6	8	4	5	7	9	10	11	12	13	14	
1	0.033	0	0	0.177	0.03	0.022	0.059	0.03	0.056	0.019	0.064	0.008	0.014	0.059	0.011946
2	0	0	0	0.29	0.068	0.013	0.059	0.067	0.108	0.088	0.158	0.129	0.088	0.059	0.00586
3	0.033	0	0	0.177	0.03	0.022	0.059	0.03	0.056	0.019	0.064	0.008	0.014	0.059	0.011946
4	0.064	0	0	0.113	0.005	0.027	0.059	0.005	0.022	0.025	0.007	0.057	0.059	0.059	0.01494
5	0.104	0	0	0.062	0	0.02	0.059	0	0.01	0.054	0.038	0.111	0.059	0.059	0.016912
6	0.152	0	0.105	0	0	0.011	0.059	0.005	0.005	0.098	0.103	0.192	0.059	0.059	0.018705
7	0.152	0	0.105	0	0	0.011	0.059	0.005	0.005	0.098	0.103	0.192	0.059	0.059	0.018705
8	0.163	0	0.319	0	0	0.041	0.059	0.049	0.087	0.194	0.167	0.233	0.059	0.059	0.019612
9	0.168	0	0.441	0	0	0.058	0.059	0.071	0.123	0.237	0.198	0.254	0.059	0.059	0.019939
10	0.174	0	0.569	0	0	0.077	0.059	0.094	0.162	0.284	0.23	0.276	0.059	0.059	0.020289
11	0.178	0	0.634	0	0	0.087	0.059	0.106	0.182	0.308	0.247	0.287	0.059	0.059	0.020472
12	0.181	0	0.701	0	0	0.098	0.059	0.118	0.204	0.334	0.264	0.299	0.059	0.059	0.020659
13	0.18	0	0.674	0	0	0.094	0.059	0.113	0.195	0.323	0.257	0.294	0.059	0.059	0.020584
14	0.178	0	0.634	0	0	0.087	0.059	0.106	0.182	0.308	0.247	0.287	0.059	0.059	0.020472
15	0.174	0	0.569	0	0	0.077	0.059	0.094	0.162	0.284	0.23	0.276	0.059	0.059	0.020289
16	0.171	0	0.504	0	0	0.068	0.059	0.082	0.142	0.26	0.214	0.265	0.059	0.059	0.020111
17	0.163	0	0.319	0	0	0.041	0.059	0.049	0.087	0.194	0.167	0.233	0.059	0.059	0.019612
18	0.162	0	0.308	0	0	0.039	0.059	0.047	0.084	0.19	0.164	0.231	0.059	0.059	0.019581
19	0.163	0	0.319	0	0	0.041	0.059	0.049	0.087	0.194	0.167	0.233	0.059	0.059	0.019612
20	0.157	0	0.207	0	0	0.025	0.059	0.026	0.045	0.145	0.135	0.212	0.059	0.059	0.019241
21	0.155	0	0.153	0	0	0.018	0.059	0.016	0.024	0.121	0.119	0.202	0.059	0.059	0.019079
22	0.135	0	0.014	0.024	0	0.013	0.059	0	0.007	0.071	0.069	0.151	0.059	0.059	0.01799
23	0.104	0	0	0.062	0	0.02	0.059	0	0.01	0.054	0.038	0.111	0.059	0.059	0.016912
24	0.076	0	0	0.097	0	0.027	0.059	0	0.014	0.037	0.009	0.073	0.059	0.059	0.015809

Based on above tables and figures, we concluded that:

1. From the Fig. 3.8, we can see that active power marginal prices are in the same order for the different buses and their daily changes have the same shape as the daily load percentage change.
2. RPMPs on various buses have quite different values and also they have quite different contours as compared with that of the daily load percentage change.

3.3.1.4 Impact of voltage control (Case 5)

From the above discussions, we came to conclusion that the bus #3 has the most serious voltage problem. Hence in this case, the voltage level of bus #3 is controlled and varied from 0.92 to 1.00 (bus #1 will not keep constant voltage of 1.06pu) and impact of voltage control of bus #3 on voltage profiles of other buses, RPMPs and reactive power output of generators and capacitors have been studied and results are described in below tables and figures.

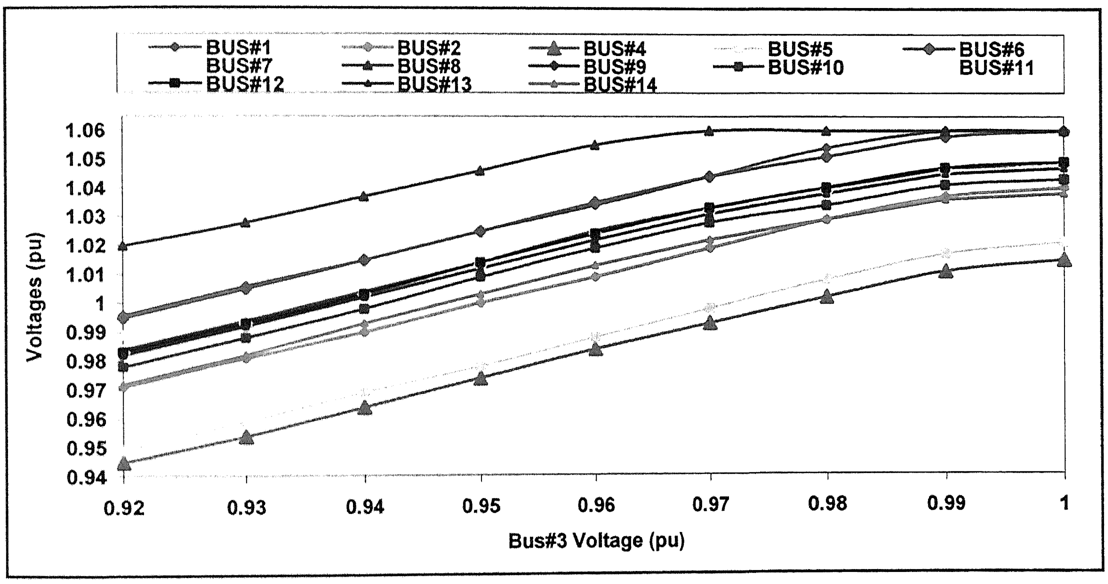


Fig. 3.10: Voltage control- voltage profiles of IEEE-14 bus system

Table 3.7: Voltage control - voltage profiles of IEEE-14 bus system

Bus No.	Voltages magnitudes (p.u.)								
	V3=0.92	V3=0.93	V3=0.94	V3=0.95	V3= 0.96	V3= 0.97	V3=0.98	V3=0.99	V3=1.0
1	0.996	1.006	1.015	1.025	1.034	1.044	1.054	1.06	1.06
2	0.971	0.981	0.99	1	1.009	1.019	1.029	1.037	1.04
4	0.945	0.954	0.964	0.974	0.984	0.993	1.002	1.011	1.015
5	0.949	0.959	0.969	0.978	0.988	0.998	1.008	1.017	1.021
6	0.995	1.005	1.015	1.025	1.035	1.044	1.051	1.058	1.06
7	0.989	0.999	1.009	1.019	1.028	1.036	1.041	1.046	1.049
8	1.02	1.028	1.037	1.046	1.055	1.06	1.06	1.06	1.06
9	0.984	0.994	1.004	1.014	1.025	1.033	1.04	1.046	1.048
10	0.978	0.988	0.998	1.009	1.019	1.028	1.034	1.041	1.043
11	0.983	0.993	1.003	1.013	1.024	1.032	1.039	1.046	1.048
12	0.983	0.993	1.003	1.014	1.024	1.033	1.04	1.047	1.049
13	0.982	0.992	1.002	1.012	1.022	1.031	1.038	1.045	1.047
14	0.972	0.982	0.993	1.003	1.013	1.022	1.029	1.036	1.038

Table 3.8: Voltage control-reactive power output of generators and capacitors of IEEE-14 bus system

Gen. Bus No.	Generator reactive power output (MVAr)								
	V3=0.92	V3=0.93	V3=0.94	V3=0.95	V3=0.96	V3=0.97	V3=0.98	V3=0.99	V3=1.0
1	10	10	10	10	10	10	10	5.83	0
2	27.18	26.69	26.19	25.7	25.21	25.04	25.3	26.46	24.96
3	0	0	0	0	0	0	0	1.19	7.94
6	6.66	6.45	6.25	6.05	5.86	5.03	3.4	2.38	1.31
8	17.72	17.32	16.92	16.53	16.13	14.46	11.22	8.21	6.9
Cap. bus	Capacitor reactive power output (MVAr)								
5	5.8163	5.5419	5.2658	4.9808	4.695	5.9255	9.0704	13.3596	15.6129
13	5.5603	5.5258	5.4915	5.4573	5.4227	5.3929	5.3711	5.3496	5.3377
14	5.3363	5.3128	5.2896	5.2668	5.2444	5.2851	5.4038	5.6496	5.6307

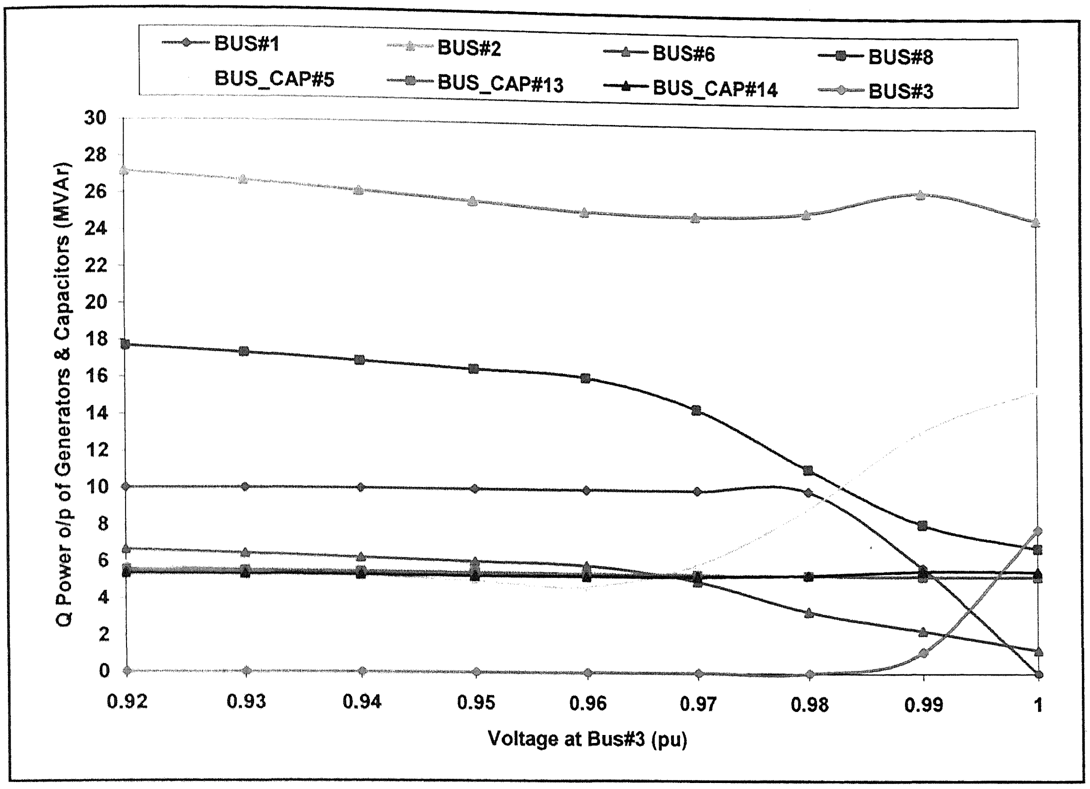


Fig. 3.11: Voltage control- reactive power o/p of generators and capacitors of IEEE-14 bus system

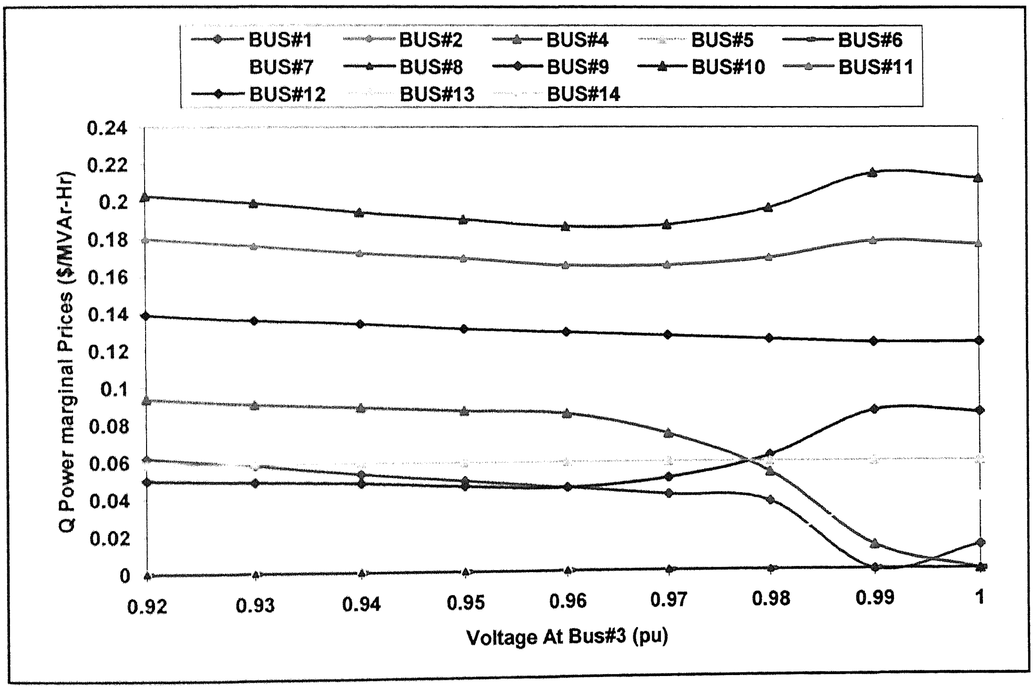


Fig. 3.12: Voltage control- reactive power marginal prices of IEEE-14 bus system

Table 3.9: Voltage control- reactive power marginal prices of IEEE-14 bus system

Bus	Reactive power marginal prices ($/MVAr-hr)								
No.	V3=0.92	V3=0.93	V3=0.94	V3=0.95	V3=0.96	V3=0.97	V3=0.98	V3=0.99	V3=1.0
1	0.062	0.058	0.053	0.049	0.045	0.041	0.037	0	0.013
2	0	0	0	0	0	0	0	0	0
3	0.24	0.228	0.217	0.207	0.197	0.182	0.159	0	0
4	0.094	0.091	0.089	0.087	0.085	0.074	0.053	0.013	0
5	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0.005	0.016	0.036	0.039
8	0	0	0	0	0	0	0	0	0
9	0.05	0.049	0.048	0.046	0.045	0.05	0.062	0.086	0.085
10	0.203	0.199	0.194	0.19	0.186	0.187	0.196	0.215	0.212
11	0.18	0.176	0.172	0.169	0.165	0.165	0.169	0.178	0.176
12	0.139	0.136	0.134	0.131	0.129	0.127	0.125	0.123	0.123
13	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059
14	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.059

Based on above results, we concluded that:

1. System voltage profiles changes simultaneously with the voltage change of bus #3 and some buses have similar voltage contours as bus #3.
2. When voltage of bus #1, #6 and #8 reaches its upper limits, generators at that buses start observe reactive power output in order to keep the voltage not to exceeding its limit values.
3. When the buses reach their voltage limits, the Q power output of generators & capacitors start fluctuates due to which significant changes in RPMPs can be observed.

3.3.2 IEEE-118 Bus System

The IEEE-118 bus system has been taken from the reference [18]. From Chapter 2, we concluded that out of 64 load buses only 44 load buses of IEEE-118 bus system were selected for installation of capacitor units. Therefore, capacitor units are placed in those selected buses.

In IEEE-118 bus system, only four cases have been studied to observe the impact on RPMPs and other system parameters. The results of the studies conducted on this system are discussed below in brief (plots are shown only for few buses for demonstration).

3.3.2.1 Impact of change of objective function (case 1 and 2)

The results obtained from OPF model for case 1 and case 2 are described in Table 3.10 based on that we noticed that similar variations have been observed as in IEEE-14 bus system except the following point:

1. In IEEE-14 bus system, the active power marginal prices at various buses are in the same order but it is not true with large system (IEEE-118) because active power marginal depend on electrical distances.

Table 3.10: Comparison of results of IEEE-118 bus system for case 1 & case 2

	Case 1	Case 2
Objective function	$COST = \sum_{i \in NG} C_i(P_{gi}) + \sum_{i \in NL} Q_{ci}.CAPCOST$	$COST = \sum_{i \in NG} C_i(P_{gi})$
Total active power production cost of generators	\$129633.68/hr	\$129629.42/hr
Total capital cost of capacitors	\$10.0243/hr	-----
ρ_{q_avg}	\$0.006971/MVAr-Hr	-----

Bus No.	Generators output (MVA)				Output of Capacitor units (MVAr)		Active & Reactive power Marginal prices			
	Case 1		Case 2		Case 1	Case 2	Case 1		Case 2	
	Real power	Reactive power	Real power	Reactive power	Reactive power	Reactive power	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)
1	26.15	15	26.03	15			40.523	0.146	40.521	0.098
4	0	62.55	0	55.43			39.337	-	39.339	-
6	0	32.49	0	32.05			39.974	-	39.974	-
8	0	-85.12	0	-90.21			39.234	-	39.237	-
10	401.95	-97.85	402	-96.4			37.864	-	37.867	-
12	85.79	31.39	85.79	8.66			40.186	-	40.185	-
15	20.86	19.64	20.82	16.03			40.417	-	40.416	-
18	13.23	29.79	13.21	29.33			40.265	-	40.264	-
19	21.51	21.53	21.47	19.36			40.43	-	40.429	-
24	0	-8.63	0	-9.16			39.298	-	39.3	-
25	193.85	-47	193.87	-47			37.622	0.132	37.625	0.131
26	279.8	-28.69	279.84	-29.08			37.822	-	37.824	-
27	9.97	25.51	10.03	21.35			40.199	-	40.201	-
31	7.25	27.23	7.25	22.8			40.71	-	40.711	-
32	14.91	21.22	14.94	16.7			40.298	-	40.299	-

34	4.93	-8	4.82	-8			40.099	0.008	40.096	0.024
36	10.7	18.95	10.61	14.25			40.214	-	40.212	-
40	49.35	26.89	49.27	16.83			40.987	-	40.985	-
42	41.06	21.85	41.09	19.9			40.821	-	40.822	-
46	19.04	-4.9	19.04	-6.82			40.045	-	40.047	-
49	193.37	15.16	193.4	9.6			38.958	-	38.961	-
54	49.54	27.16	49.54	24.36			40.642	-	40.643	-
55	32.13	19.27	32.14	19.3			40.643	-	40.643	-
56	32.55	13.84	32.55	10.93			40.651	-	40.651	0
59	149.72	95.31	149.72	87.69			39.319	-	39.319	-
61	148.44	32.07	148.45	26.1			38.555	-	38.556	-
62	0	1.04	0	0.73			38.748	-	38.75	-
65	352.31	-67	352.36	-67			38.021	0.005	38.023	0.006
66	348.93	-67	348.99	-67			37.803	0.212	37.805	0.211
69	453.81	-118.55	453.88	-121.2			37.576	-	37.579	-
70	0	17.24	0	13.82			39.725	-	39.726	-
72	0	-5.2	0	-5.2			39.744	-	39.745	-
73	0	-2.22	0	-2.22			39.799	-	39.8	-
74	16.34	9	16.19	9			40.327	0.073	40.324	0.027
76	21.93	23	21.76	23			40.439	0.164	40.435	0.12
77	0	51.29	0	37.66			38.945	-	38.946	-
80	430.94	-27.82	431	-42.63			38.069	-	38.071	-
85	0	23	0	21.02			37.78	0.037	37.783	0
87	3.63	0.57	3.63	-2.44			38.132	-	38.13	-
89	502.43	-43.02	502.74	-55.58			36.555	-	36.565	-
90	0	47.24	0	47.26			38.304	-	38.314	-
91	0	-0.69	0	-1.72			38.14	-	38.148	-
92	0	9	0	9			37.623	0.049	37.631	0.022
99	0	-3.18	0	-3.18			38.651	-	38.652	-

100	231.31	20.78	231.33	9.29			38.358	-	38.36	-
103	38.25	11.69	38.25	11.68			39.124	-	39.124	-
104	0	23	0	23			39.886	0.004	39.886	0.004
105	5.13	5.89	5.11	0.23			40.103	-	40.102	-
107	28.98	2.39	28.91	0.9			40.58	-	40.578	-
110	7.03	19.66	7.03	19.65			40.141	-	40.141	-
111	35.24	-0.25	35.24	-0.25			39.579	-	39.579	-
112	36.47	10.28	36.46	10.28			40.729	-	40.729	-
113	0	-9.99	0	-10.61			39.786	-	39.786	-
116	0	6.81	0	7.26			37.995	-	37.998	-
2							40.487	0.117	40.485	0.098
5							39.225	0.059	39.228	0.056
7							40.091	0.003	40.091	0.003
9							38.557	0.133	38.559	0.131
17							39.673	0.043	39.673	0.047
20							40.532	0.037	40.528	0.003
22							39.937	0.089	39.938	0.05
23							39.004	0.014	39.007	0.009
28							40.605	0.061	40.605	0.045
30							39.461	0.018	39.462	0.023
35							40.207	0.006	40.205	0.002
37							39.913	0.035	39.911	0.056
48							39.403	0.079	39.406	0.079
64							38.598	0.024	38.6	0.024
68							37.949	0.003	37.952	0.004
78							39.048	0.064	39.048	0.04
81							37.988	0.016	37.99	0.016
108							40.209	0.008	40.208	0.008
109							40.235	0.015	40.235	0.015

115					40.424	0.047	40.424	0.009
3			16.7312	22.6957	40.297	0.059	40.296	-
11			7.6213	28.8177	40.129	0.059	40.128	-
13			11.1926	12.2111	40.631	0.059	40.626	-
14			0	0	40.409	0.03	40.409	0.03
16			0	5.3794	40.286	0.059	40.284	-
21			6.9917	10.079	40.358	0.059	40.355	-
29			0	5.352	40.809	0.035	40.809	0
33			0.4298	3.7405	40.514	0.059	40.511	-
38			0	2.9853	39.667	0.014	39.667	-
39			1.6979	7.5153	40.828	0.059	40.825	-
41			0.8027	7.9663	41.248	0.059	41.246	-
43			0	0	40.842	0.007	40.837	0.024
44			0	0	41.124	0.044	41.118	0.096
45			1.2923	4.5577	40.949	0.059	40.946	-
47			0	0	39.359	0.016	39.362	0.016
50			0	3.1551	39.696	0.052	39.697	-
51			5.8651	8.264	40.654	0.059	40.653	-
52			2.5518	2.4488	41.002	0.059	40.999	-
53			4.8337	7.6173	41.081	0.059	41.078	-
57			0	0.8107	40.43	0.034	40.429	0
58			0	0.8311	40.788	0.043	40.787	-
60			0	2.2645	38.747	0.003	38.749	-
63			0	12.3272	38.965	0.004	38.965	-
67			0	0	38.467	0.063	38.47	0.063
71			0	0	39.761	0.01	39.762	0.01
75			6.0941	12.6237	40.027	0.059	40.025	-
79			7.9365	21.9799	38.904	0.059	38.904	-
82			0	0	39.112	0.002	39.108	0.044

83			0	0	38.89	0.017	38.886	0.06
84			2.8494	4.2121	38.296	0.059	38.295	-
86			2.521	6.3587	38.203	0.059	38.201	-
88			5.3908	13.2881	37.321	0.059	37.327	-
93			5.7726	7.5322	38.271	0.059	38.274	-
94			5.4408	11.063	38.648	0.059	38.647	-
95			30.0851	30.1162	38.976	0.059	38.972	-
96			0	3.1441	38.887	0.058	38.883	-
97			3.0946	7.7372	38.579	0.059	38.579	-
98			0	4.5907	38.528	0.048	38.528	0
101			9.0963	12.1105	38.304	0.059	38.305	-
102			0.3829	4.1259	37.878	0.059	37.883	-
106			4.8432	13.2109	40.214	0.059	40.212	-
114			0	7.3364	40.418	0.043	40.418	-
117			4.1811	6.561	40.678	0.059	40.673	-
118			22.4927	23.7771	40.414	0.059	40.41	-

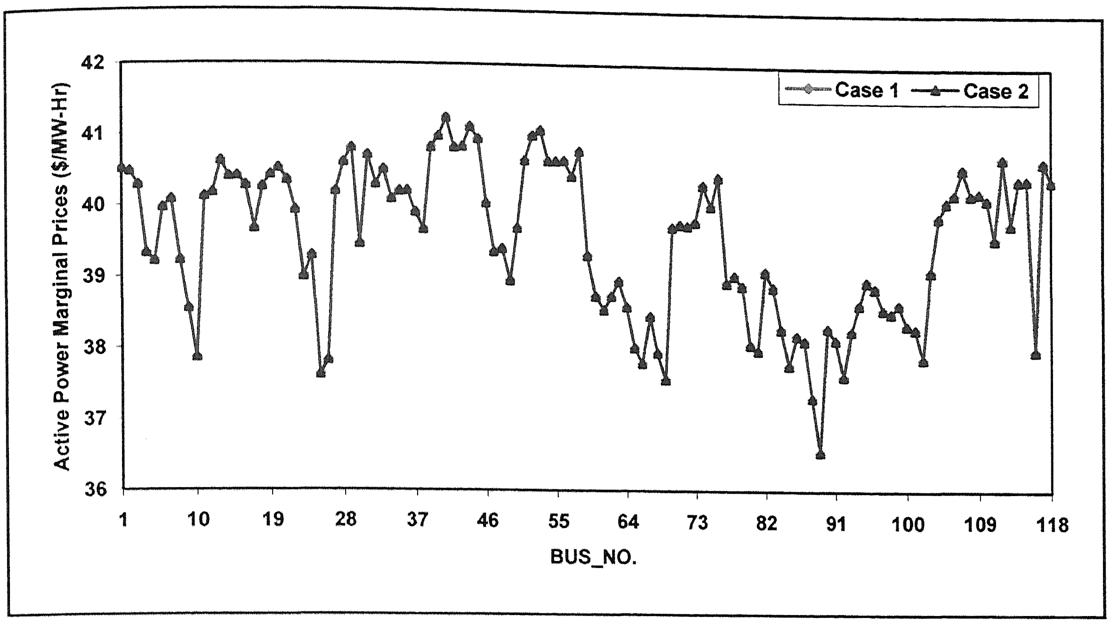


Fig. 3.13: Comparison of active power marginal prices of IEEE-118 bus system for case 1 & case 2

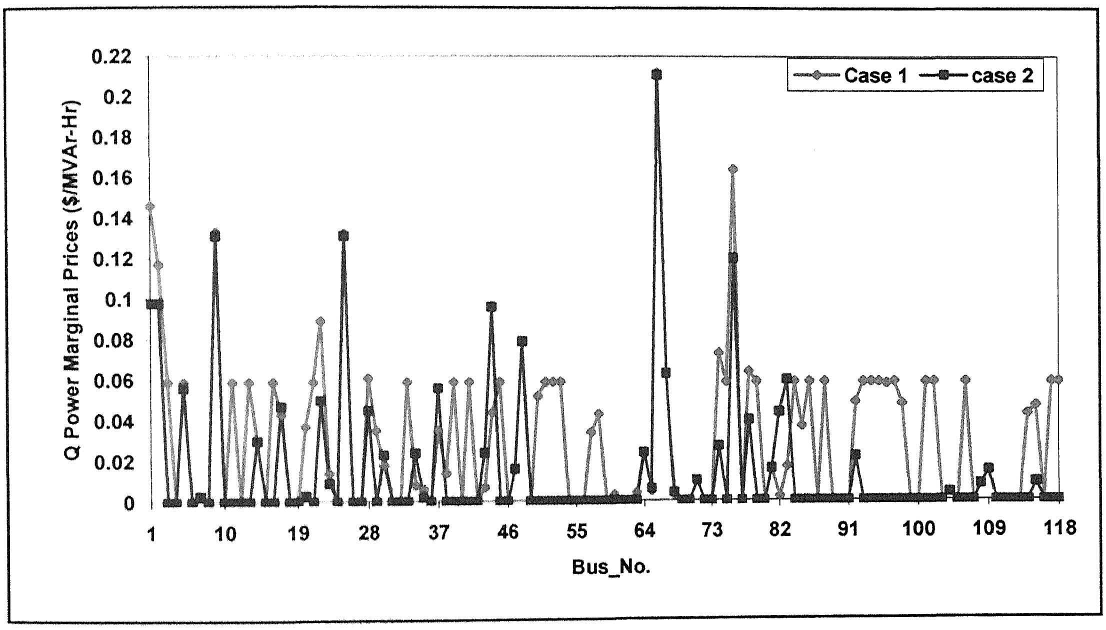


Fig. 3.14: Comparison of reactive power marginal prices of IEEE-118 bus system for case 1 & case 2

3.3.2.2 Impact of load power factor (case 3)

In this case, the power factor of the load is varied from 0.7 to 0.95 and it's impact on RPMPs, voltage profiles and reactive power output of generators and capacitors have been studied and results are described in below tables and figures.

Table 3.11: Load pf - reactive power marginal prices of IEEE-118 bus system

Bus No.	Reactive power marginal prices ($/MVAr-Hr)					
	Pf=0.7	Pf=0.75	Pf=0.8	Pf=0.85	Pf=0.9	Pf=0.95
1	0.381	0.315	0.253	0.192	0.13	0.06
2	0.308	0.258	0.21	0.164	0.116	0.063
3	0.059	0.059	0.059	0.059	0.059	0.059
4	0	0	0	0	0	0
5	0.061	0.06	0.059	0.058	0.059	0.06
6	0.115	0.073	0.034	0	0	0
7	0.117	0.084	0.052	0.025	0.019	0.012
8	0	0	0	0	0	0
9	0.151	0.146	0.14	0.136	0.135	0.133
10	0	0	0	0	0	0
11	0.059	0.059	0.059	0.059	0.059	0.059
12	0	0	0	0	0	0
13	0.059	0.059	0.059	0.059	0.059	0.059
14	0.059	0.059	0.059	0.059	0.043	0.012
15	0.384	0.276	0.174	0.089	0.021	0
16	0.059	0.059	0.059	0.059	0.059	0.041
17	0.082	0.046	0.011	0.017	0.031	0.04
18	0.289	0.169	0.057	0	0	0
19	0.381	0.255	0.137	0.05	0	0
20	0.327	0.252	0.181	0.125	0.083	0.061
21	0.059	0.059	0.059	0.059	0.059	0.059
22	0.152	0.135	0.118	0.103	0.088	0.071
23	0.044	0.035	0.026	0.02	0.015	0.009
24	0	0	0	0	0	0
25	0.161	0.153	0.145	0.138	0.135	0.132
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0.158	0.138	0.119	0.101	0.082	0.06
29	0.059	0.059	0.059	0.059	0.059	0.057
30	0.055	0.036	0.015	0.004	0.012	0.02
31	0	0	0	0	0	0
32	0.073	0.04	0.01	0	0	0

33	0.059	0.059	0.059	0.059	0.059	0.033
34	0.104	0.073	0.033	0	0	0.038
35	0.233	0.179	0.117	0.052	0.018	0.004
36	0.217	0.162	0.099	0.035	0.002	0
37	0.036	0.03	0.011	0.024	0.029	0.058
38	0.059	0.059	0.051	0.026	0.021	0
39	0.059	0.059	0.059	0.059	0.059	0.046
40	0	0	0	0	0	0
41	0.059	0.059	0.059	0.059	0.059	0.059
42	0	0	0	0	0	0
43	0.059	0.059	0.059	0.059	0.047	0.089
44	0.059	0.05	0.022	0.005	0.036	0.184
45	0.059	0.059	0.059	0.059	0.059	0.048
46	0	0	0	0	0	0
47	0.059	0.059	0.059	0.059	0.059	0.059
48	0.017	0.012	0.039	0.065	0.092	0.123
49	0	0	0	0	0	0
50	0.059	0.059	0.059	0.059	0.059	0.059
51	0.059	0.059	0.059	0.059	0.059	0.059
52	0.059	0.059	0.059	0.059	0.059	0.059
53	0.059	0.059	0.059	0.059	0.059	0.059
54	0	0	0	0	0	0
55	0.237	0.186	0.134	0.085	0.042	0
56	0.161	0.13	0.098	0.067	0.04	0.012
57	0.059	0.059	0.059	0.059	0.059	0.059
58	0.059	0.059	0.059	0.059	0.059	0.059
59	0.136	0.105	0.051	0	0	0
60	0.059	0.059	0.059	0.059	0.059	0.036
61	0	0	0	0	0	0
62	0.162	0.133	0.105	0.078	0.043	0
63	0.059	0.059	0.038	0.007	0.005	0.003
64	0.012	0.003	0.01	0.022	0.023	0.026
65	0	0	0	0	0	0.01
66	0.211	0.202	0.192	0.188	0.197	0.222
67	0.059	0.059	0.059	0.059	0.021	0.05
68	0.011	0.009	0.008	0.007	0.005	0.002
69	0	0	0	0	0	0
70	0.196	0.152	0.098	0.047	0	0
71	0.059	0.058	0.034	0.011	0.01	0.01
72	0	0	0	0	0	0
73	0	0	0	0	0	0
74	0.445	0.36	0.277	0.196	0.116	0.039
75	0.059	0.059	0.059	0.059	0.059	0.059
76	0.53	0.423	0.323	0.226	0.135	0.037

77	0.135	0.093	0.054	0.015	0	0
78	0.222	0.181	0.141	0.102	0.075	0.039
79	0.059	0.059	0.059	0.059	0.052	0.007
80	0	0	0	0	0	0
81	0.034	0.029	0.025	0.022	0.019	0.013
82	0.059	0.059	0.059	0.053	0.01	0.106
83	0.059	0.059	0.059	0.037	0.027	0.124
84	0.059	0.059	0.059	0.059	0.059	0.009
85	0.086	0.072	0.059	0.044	0.022	0
86	0.059	0.059	0.059	0.059	0.059	0.035
87	0	0	0	0	0	0
88	0.059	0.059	0.059	0.059	0.059	0.059
89	0	0	0	0	0	0
90	0	0	0	0	0	0
91	0	0	0	0	0	0
92	0.179	0.158	0.138	0.118	0.098	0.074
93	0.059	0.059	0.059	0.059	0.059	0.059
94	0.059	0.059	0.059	0.059	0.059	0.052
95	0.059	0.059	0.059	0.059	0.059	0.059
96	0.059	0.059	0.059	0.059	0.059	0.016
97	0.059	0.059	0.059	0.059	0.059	0.031
98	0.059	0.059	0.059	0.059	0.059	0.059
99	0	0	0	0	0	0
100	0	0	0	0	0	0
101	0.059	0.059	0.059	0.059	0.059	0.059
102	0.059	0.059	0.059	0.059	0.059	0.059
103	0	0	0	0	0	0
104	0.084	0.048	0.022	0	0	0
105	0.011	0	0	0	0	0.01
106	0.059	0.059	0.059	0.059	0.059	0.059
107	0	0	0	0	0	0
108	0.096	0.061	0.039	0.027	0.015	0.005
109	0.125	0.082	0.054	0.04	0.025	0.003
110	0.061	0.022	0	0	0	0
111	0	0	0	0	0	0
112	0	0	0	0	0	0
113	0	0	0	0	0	0
114	0.059	0.059	0.059	0.059	0.059	0.042
115	0.085	0.08	0.075	0.071	0.066	0.047
116	0	0	0	0	0	0
117	0.059	0.059	0.059	0.059	0.059	0.059
118	0.059	0.059	0.059	0.059	0.059	0.059
Avg. cost	0.017823	0.015495	0.012888	0.0105569	0.007552	0.003839

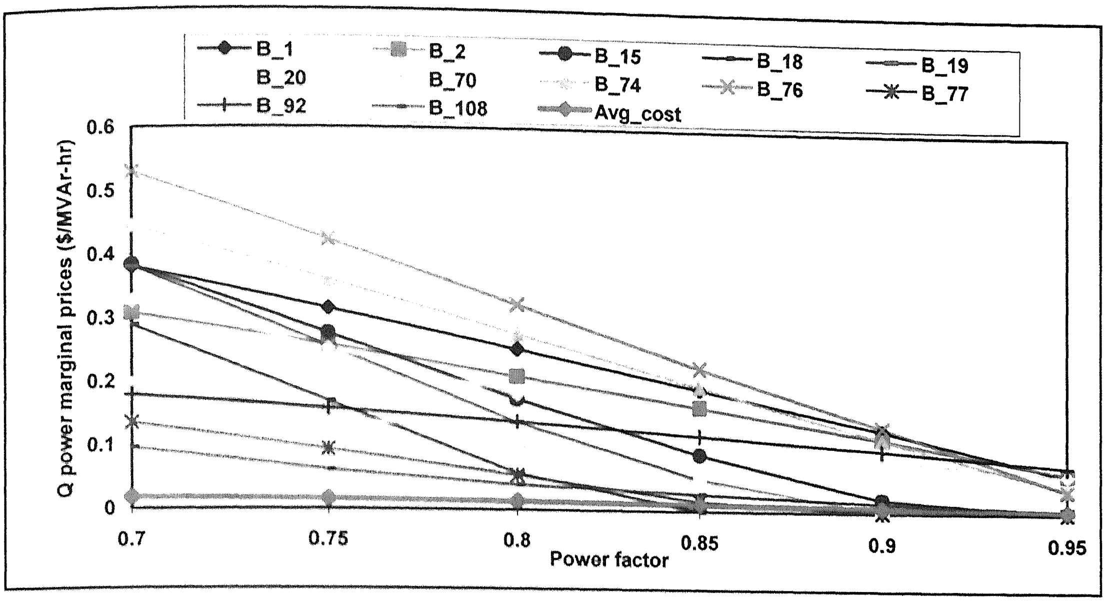


Fig. 3.15: Load pf-reactive power marginal prices and the average cost of IEEE-118 bus system

Table 3.12: Load pf- voltage magnitude profiles of IEEE-118 bus system

Bus No.	Voltage magnitudes (p.u.)					
	Pf=0.7	Pf=0.75	Pf=0.8	Pf=0.85	Pf=0.9	Pf=0.95
1	1.03	1.033	1.036	1.038	1.041	1.043
2	1.033	1.036	1.038	1.04	1.042	1.044
3	1.046	1.046	1.046	1.046	1.046	1.046
4	1.06	1.06	1.06	1.06	1.06	1.06
5	1.058	1.058	1.058	1.058	1.058	1.058
6	1.046	1.049	1.051	1.053	1.053	1.053
7	1.045	1.047	1.049	1.05	1.05	1.051
8	1.043	1.042	1.042	1.041	1.041	1.04
9	1.06	1.06	1.06	1.06	1.06	1.06
10	1.051	1.051	1.052	1.053	1.053	1.054
11	1.048	1.048	1.048	1.048	1.048	1.048
12	1.05	1.05	1.05	1.05	1.05	1.05
13	1.042	1.042	1.042	1.042	1.042	1.042
14	1.044	1.045	1.045	1.045	1.046	1.047
15	1.031	1.036	1.042	1.045	1.048	1.049
16	1.046	1.046	1.047	1.047	1.047	1.047
17	1.053	1.055	1.058	1.059	1.06	1.06
18	1.036	1.042	1.049	1.051	1.051	1.05
19	1.03	1.036	1.042	1.046	1.049	1.048
20	1.028	1.031	1.035	1.037	1.039	1.04

21	1.042	1.042	1.041	1.04	1.04	1.04
22	1.038	1.039	1.04	1.04	1.04	1.041
23	1.048	1.049	1.049	1.049	1.049	1.05
24	1.047	1.047	1.047	1.047	1.046	1.046
25	1.06	1.06	1.06	1.06	1.06	1.06
26	1.027	1.027	1.027	1.026	1.026	1.026
27	1.04	1.041	1.042	1.042	1.042	1.042
28	1.029	1.03	1.032	1.033	1.034	1.035
29	1.033	1.034	1.035	1.035	1.035	1.035
30	1.031	1.031	1.031	1.031	1.031	1.031
31	1.037	1.038	1.039	1.039	1.039	1.039
32	1.037	1.039	1.041	1.041	1.041	1.041
33	1.045	1.047	1.048	1.047	1.047	1.047
34	1.051	1.054	1.056	1.056	1.056	1.056
35	1.043	1.047	1.051	1.052	1.053	1.053
36	1.044	1.048	1.052	1.053	1.054	1.053
37	1.057	1.059	1.06	1.06	1.06	1.06
38	1.024	1.02	1.016	1.016	1.015	1.015
39	1.043	1.044	1.045	1.044	1.043	1.043
40	1.042	1.043	1.044	1.043	1.043	1.041
41	1.035	1.037	1.037	1.036	1.036	1.035
42	1.04	1.041	1.042	1.041	1.041	1.04
43	1.038	1.039	1.04	1.038	1.039	1.044
44	1.026	1.028	1.03	1.031	1.032	1.037
45	1.026	1.027	1.028	1.027	1.027	1.029
46	1.037	1.038	1.039	1.039	1.038	1.037
47	1.041	1.042	1.043	1.043	1.042	1.041
48	1.043	1.045	1.047	1.048	1.049	1.048
49	1.049	1.05	1.051	1.051	1.05	1.049
50	1.037	1.039	1.04	1.04	1.04	1.038
51	1.026	1.028	1.029	1.03	1.029	1.027
52	1.022	1.023	1.025	1.025	1.025	1.023
53	1.021	1.022	1.024	1.024	1.024	1.022
54	1.029	1.031	1.033	1.033	1.032	1.03
55	1.018	1.022	1.027	1.029	1.03	1.03
56	1.022	1.025	1.028	1.03	1.03	1.03
57	1.028	1.03	1.032	1.032	1.032	1.03
58	1.024	1.026	1.028	1.028	1.027	1.026
59	1.036	1.04	1.046	1.049	1.048	1.046
60	1.045	1.046	1.046	1.045	1.044	1.043
61	1.052	1.052	1.052	1.051	1.05	1.047
62	1.037	1.039	1.041	1.041	1.043	1.043
63	1.029	1.021	1.017	1.018	1.017	1.015
64	1.032	1.028	1.026	1.026	1.024	1.022

65	1.027	1.023	1.021	1.019	1.017	1.015
66	1.06	1.06	1.06	1.06	1.06	1.06
67	1.04	1.041	1.041	1.041	1.043	1.045
68	1.023	1.021	1.019	1.018	1.016	1.014
69	1.06	1.06	1.06	1.06	1.06	1.06
70	1.029	1.031	1.034	1.036	1.039	1.039
71	1.036	1.036	1.037	1.038	1.039	1.039
72	1.04	1.04	1.04	1.04	1.04	1.04
73	1.039	1.039	1.039	1.038	1.038	1.038
74	1.012	1.016	1.019	1.023	1.026	1.029
75	1.032	1.032	1.032	1.032	1.032	1.032
76	1.008	1.012	1.016	1.02	1.024	1.028
77	1.042	1.044	1.045	1.047	1.048	1.048
78	1.037	1.038	1.04	1.042	1.043	1.045
79	1.048	1.047	1.047	1.046	1.047	1.049
80	1.06	1.06	1.06	1.06	1.06	1.06
81	1.016	1.015	1.013	1.013	1.012	1.01
82	1.04	1.041	1.041	1.041	1.044	1.048
83	1.042	1.042	1.042	1.043	1.046	1.051
84	1.047	1.047	1.048	1.048	1.049	1.052
85	1.052	1.053	1.053	1.054	1.056	1.057
86	1.047	1.048	1.048	1.048	1.049	1.05
87	1.054	1.055	1.055	1.055	1.055	1.055
88	1.052	1.052	1.052	1.052	1.052	1.052
89	1.06	1.06	1.06	1.06	1.06	1.06
90	1.042	1.042	1.042	1.042	1.042	1.042
91	1.045	1.045	1.045	1.046	1.046	1.046
92	1.046	1.047	1.048	1.05	1.051	1.052
93	1.048	1.048	1.048	1.048	1.049	1.049
94	1.047	1.047	1.047	1.047	1.047	1.048
95	1.043	1.043	1.043	1.043	1.043	1.043
96	1.044	1.044	1.044	1.044	1.044	1.047
97	1.049	1.049	1.049	1.049	1.049	1.051
98	1.051	1.051	1.051	1.051	1.051	1.051
99	1.054	1.054	1.054	1.054	1.054	1.054
100	1.059	1.059	1.059	1.059	1.059	1.059
101	1.051	1.051	1.051	1.051	1.051	1.051
102	1.052	1.052	1.052	1.052	1.052	1.052
103	1.051	1.051	1.051	1.052	1.052	1.052
104	1.039	1.04	1.042	1.043	1.043	1.043
105	1.04	1.04	1.04	1.041	1.041	1.041
106	1.036	1.036	1.036	1.036	1.036	1.036
107	1.034	1.034	1.034	1.034	1.034	1.035
108	1.036	1.037	1.038	1.039	1.039	1.04

109	1.035	1.037	1.038	1.038	1.039	1.04
110	1.039	1.04	1.042	1.042	1.042	1.042
111	1.049	1.049	1.049	1.049	1.049	1.05
112	1.033	1.034	1.034	1.034	1.034	1.035
113	1.054	1.055	1.056	1.056	1.056	1.056
114	1.035	1.036	1.036	1.036	1.036	1.037
115	1.034	1.034	1.035	1.035	1.035	1.036
116	1.025	1.022	1.02	1.018	1.016	1.014
117	1.041	1.041	1.041	1.041	1.041	1.041
118	1.028	1.027	1.027	1.027	1.028	1.028

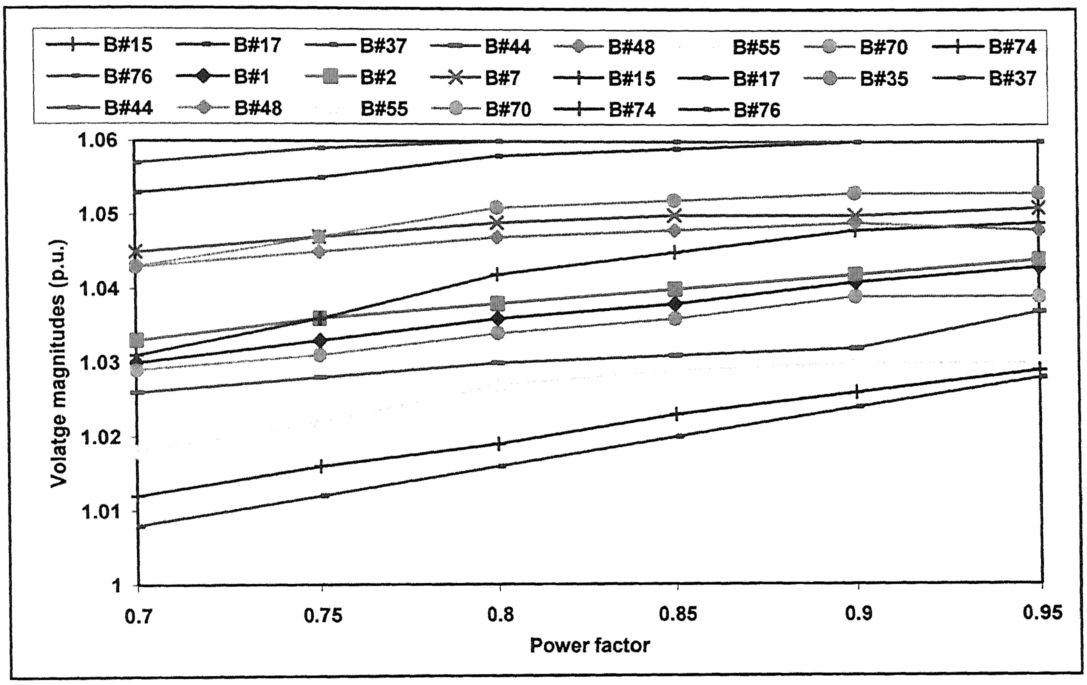


Fig. 3.16: Load pf- voltage magnitude profiles of IEEE-118 bus system

Table 3.13: Load pf- reactive power output of generators and capacitors of IEEE-118 bus system

Gen. bus No.	Generator reactive power output (MVAr)					
	Pf = 0.7	Pf = 0.75	Pf = 0.8	Pf = 0.85	Pf = 0.9	Pf = 0.95
1	15	15	15	15	15	15
4	91.55	84.8	78.66	73.15	68.44	63.69
6	50	50	50	49.03	40.36	30.39
8	-30.06	-41.73	-53.27	-64.08	-70.02	-76.6
10	-107.19	-104.41	-101.55	-99.26	-98.55	-97.52

12	106.84	91.49	76.92	63.03	51.3	35.9
15	30	30	30	30	30	19.51
18	50	50	50	41.93	26.19	15.83
19	24	24	24	24	23.23	11.93
24	12.79	8.98	5	1.32	-2.27	-4.86
25	-47	-47	-47	-47	-47	-47
26	-25.71	-27.11	-28.16	-28.67	-29.08	-29.34
27	97.88	84.75	72.27	61.05	49.83	35.71
31	53.81	45.38	37.33	30.47	24.25	17.08
32	42	42	42	36.59	28.37	17.3
34	24	24	24	10.47	-6.27	-8
36	24	24	24	24	24	6.59
40	72.85	63.59	54.45	45.01	35.98	24.14
42	96.83	83.57	70.86	58.21	45.26	30.38
46	18.52	13.97	9.64	5.39	1	-7.53
49	86.66	74.18	62.39	49.57	34.28	12.63
54	207.63	172.81	137.86	103.67	72.31	37.92
55	23	23	23	23	23	21.13
56	15	15	15	15	15	15
59	180	180	180	160.14	121.19	74.98
61	80.36	89.08	88	75.49	70.42	48.97
62	20	20	20	20	20	16.6
65	-12	-23.36	-36.58	-50.78	-63.73	-67
66	-67	-67	-67	-67	-67	-67
69	-129.07	-126.03	-124.22	-124.22	-122.1	-114.48
70	32	32	32	32	30.65	17.8
72	9.14	7.46	5.15	2.88	0.61	-1.26
73	14.7	13.77	9.21	4.78	0.69	-0.25
74	9	9	9	9	9	9
76	23	23	23	23	23	23
77	70	70	70	70	55.53	25.1
80	79.99	61.47	43.39	23.96	6.7	-16.27
85	23	23	23	23	23	17.38
87	0.56	0.57	0.57	0.57	0.57	-0.63
89	-19.15	-23.12	-26.98	-30.98	-35.13	-40.16
90	171.49	148.95	127.46	106.23	84.16	58.81
91	14.34	12.18	10.12	8.09	5.98	3.54
92	9	9	9	9	9	9
99	39.67	33.87	28.32	22.85	17.16	10.63
100	46.03	39.41	33.33	27.42	21.98	15.22
103	23.81	18.02	13.48	9.81	6.7	2.87
104	23	23	23	22.38	17.24	10.01
105	23	15.9	7.21	-0.48	-5.31	-8
107	41.69	34.49	27.89	21.37	14.6	6.58

110	23	23	20.85	15.05	9.04	1.93
111	3.54	1.14	-0.25	-0.25	-0.25	-0.25
112	69.93	58.45	48.28	39.42	30.21	19.63
113	16.89	9.89	3.07	-2.34	-5.27	-7.6
116	224.25	191.72	161.14	133.14	103.66	61.46
Cap. Bus	Capacitor reactive power output (MVAr)					
3	70.7773	58.5811	47.0232	35.638	23.9662	10.6239
11	56.2025	46.4375	37.1368	27.9404	18.4928	7.6451
13	36.7241	30.1132	23.8186	17.8595	12.0348	6.3689
14	14.2513	9.9171	5.7965	2.0514	0	0
16	19.2939	14.7458	10.385	6.2575	2.4522	0
21	38.1213	29.7135	21.7379	14.8813	9.0195	4.099
29	21.6744	17.2875	13.1041	8.9617	4.6682	0
33	30.6625	23.5121	16.297	9.157	3.5008	0
38	48.0593	22.6015	0	0	0	0
39	21.2566	17.2242	12.8468	7.854	4.0145	0
41	28.5574	23.4123	18.5182	13.7206	8.7153	2.981
43	14.4564	10.7865	6.4946	2.5041	0	0
44	1.5813	0	0	0	0	0
45	39.1851	31.3048	22.7085	14.341	5.391	0
47	25.4156	20.6883	16.1804	11.7451	7.1498	1.8898
50	13.4887	11.1207	8.8604	6.639	4.346	1.7186
51	16.0311	13.6575	11.3893	9.1634	6.8729	4.2528
52	15.9135	13.4011	11.0017	8.6513	6.2389	3.4732
53	17.3176	14.1061	11.0408	8.0383	4.9446	1.403
57	13.9377	11.0914	8.305	5.5789	2.9339	0
58	13.9178	11.0837	8.3112	5.5958	2.9589	0.0198
60	62.0159	46.6822	31.084	15.7617	1.3249	0
63	85.6606	33.6043	0	0	0	0
67	16.087	11.2486	6.7303	1.8622	0	0
71	6.323	0	0	0	0	0
75	91.374	73.7302	56.2771	39.1667	22.4944	6.9239
79	55.7804	39.8555	24.7546	9.8207	0	0
82	28.5282	19.1067	10.1136	0	0	0
83	6.1851	3.0035	0	0	0	0
84	11.1557	8.9636	6.873	4.197	0.2152	0
86	15.9885	12.5046	9.1853	5.7988	2.0549	0
88	47.9827	40.3158	33.0107	25.6072	17.522	8.4365
93	18.2821	15.4498	12.7544	10.0956	7.3292	3.7147
94	23.9686	19.1903	14.6374	10.1413	5.4663	0
95	41.9858	36.1772	30.6375	25.1665	19.475	8.7017
96	28.6188	23.4083	18.4465	13.0428	2.465	0
97	9.4603	7.383	5.4003	3.4406	1.4075	0
98	25.6316	20.929	16.4438	12.0118	7.4082	2.1172

101	16.5424	13.5001	10.5984	7.7316	4.7511	1.3261
102	17.824	14.6528	11.643	8.6787	5.5961	2.025
106	33.8245	26.7832	21.1038	15.4955	9.6698	2.0583
114	23.3427	16.3328	9.6701	5.1637	1.4616	0
117	16.5878	13.8195	11.1794	8.5737	5.8658	2.7557
118	69.5435	56.7481	44.6674	32.8304	21.2134	8.3625

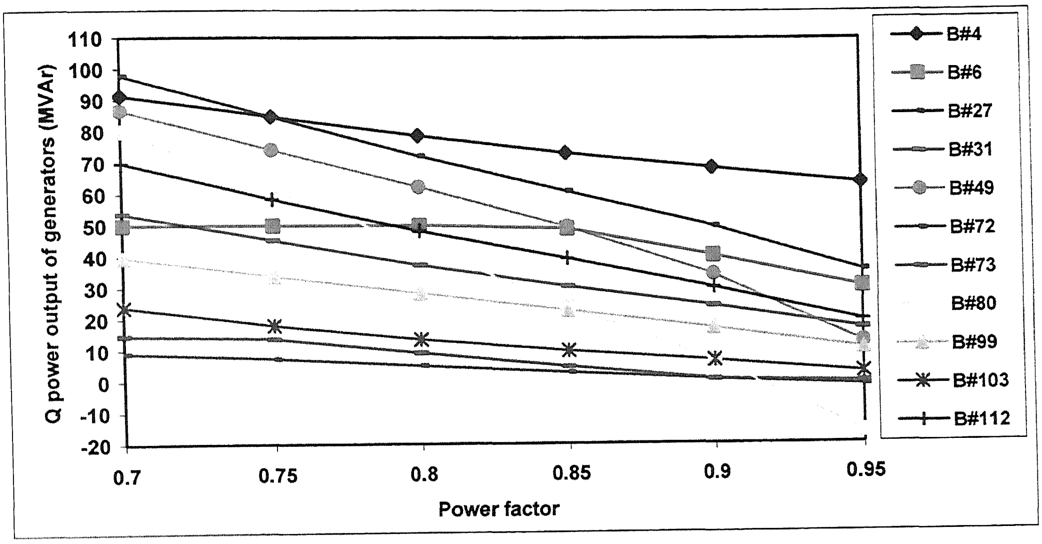


Fig. 3.17: Load pf- reactive power output of generators of IEEE-118 bus system

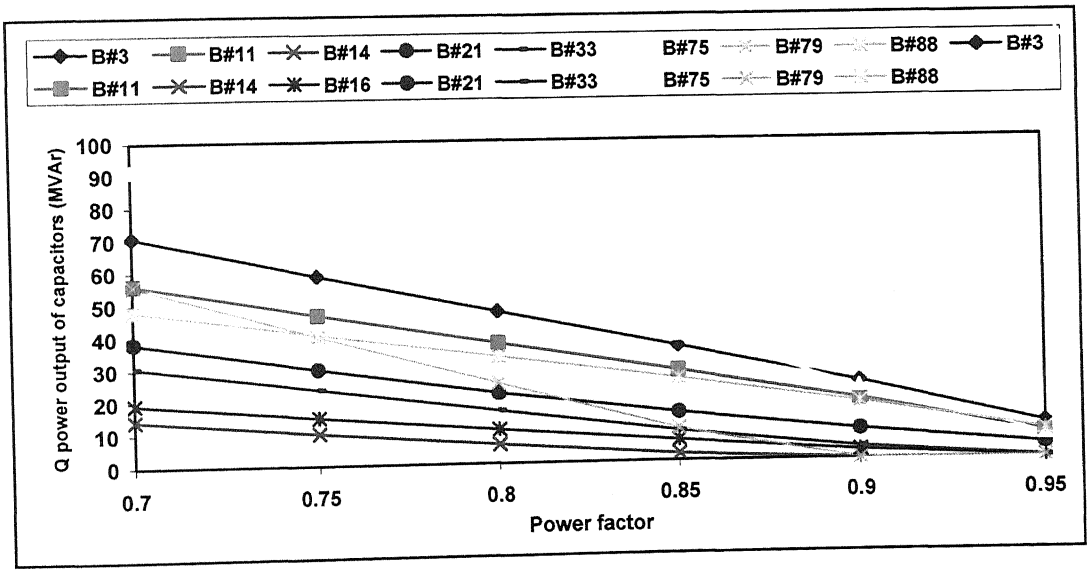


Fig. 3.18: Load pf- reactive power output of capacitor units of IEEE-118 bus system

3.3.2.3 Impact of change in system operating point (case 4)

In this case, only three loading conditions are considered in the analysis (base, minimum and maximum) corresponding to load scaling factors equal to 1.0, 0.7 and 1.4, respectively and their impact on system voltage profiles and marginal prices have been studied. The results are given in below tables and figures.

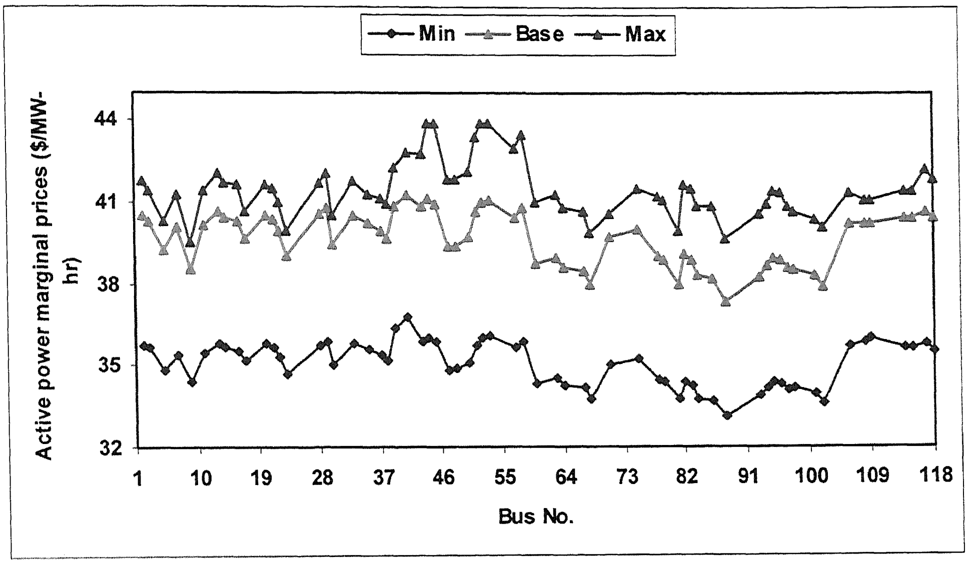


Fig. 3.19: Active power marginal prices of IEEE-118 for three load conditions

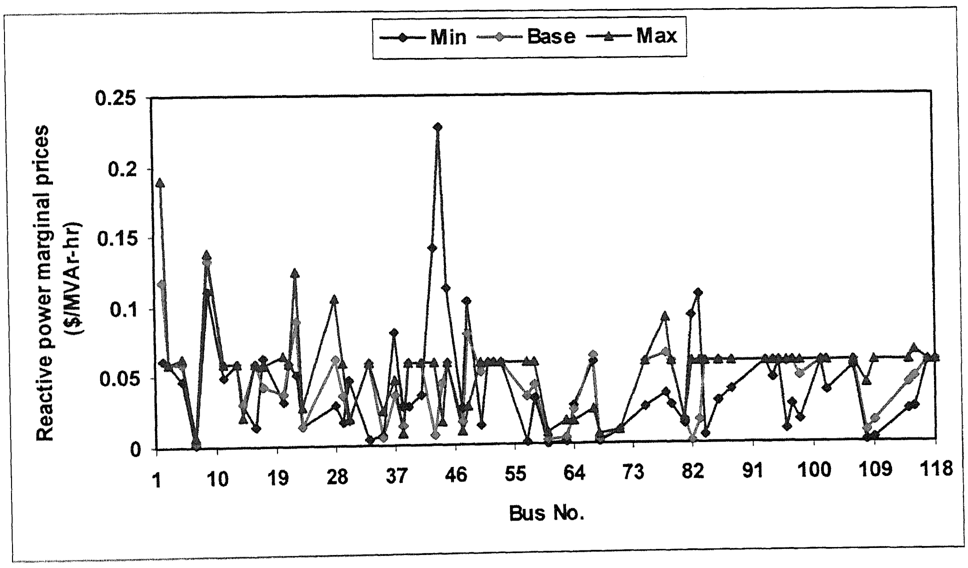


Fig 3.20: Reactive power marginal prices of IEEE-118 for three load conditions

Table 3.14: Voltage profiles and marginal prices for three load conditions of IEEE-118 bus system

Bus No.	Voltage magnitude (p.u)			Active & Reactive power marginal prices					
				LSF = 0.7(Min)		LSF = 1(Base)		LSF = 1.4(Peak)	
	LSF = 0.7 (Min)	LSF = 1 (Base)	LSF = 1.4 (Peak)	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)	MC_p (\$/MW-Hr)	MC_q (\$/MVAr-Hr)
1	1.041	1.04	1.038	35.873	0.073	40.523	0.146	41.504	0.246
2	1.043	1.042	1.037	35.726	0.061	40.487	0.117	41.74	0.19
3	1.045	1.046	1.046	35.629	0.059	40.297	0.059	41.442	0.059
4	1.058	1.06	1.06	34.889	0	39.337	0	40.415	0
5	1.056	1.058	1.058	34.816	0.046	39.225	0.059	40.29	0.063
6	1.052	1.053	1.054	35.305	0	39.974	0	41.034	0
7	1.051	1.051	1.051	35.388	0.002	40.091	0.003	41.25	0.007
8	1.038	1.04	1.041	34.834	0	39.234	0	40.281	0
9	1.06	1.06	1.06	34.365	0.11	38.557	0.133	39.55	0.138
10	1.05	1.054	1.055	33.882	0	37.864	0	38.803	0
11	1.048	1.048	1.046	35.417	0.049	40.129	0.059	41.446	0.059
12	1.05	1.05	1.048	35.466	0	40.186	0	41.478	0
13	1.043	1.042	1.038	35.762	0.059	40.631	0.059	42.069	0.059
14	1.049	1.049	1.046	35.641	0.03	40.409	0.03	41.691	0.021
15	1.048	1.049	1.048	35.75	0	40.417	0	41.394	0.019
16	1.049	1.046	1.044	35.537	0.013	40.286	0.059	41.595	0.059
17	1.059	1.06	1.06	35.181	0.063	39.673	0.043	40.647	0.057
18	1.049	1.05	1.052	35.657	0	40.265	0	41.108	0
19	1.047	1.048	1.048	35.793	0	40.43	0	41.297	0.039
20	1.042	1.041	1.04	35.785	0.031	40.532	0.037	41.614	0.064
21	1.041	1.04	1.039	35.625	0.059	40.358	0.059	41.489	0.059
22	1.044	1.04	1.038	35.308	0.05	39.937	0.089	41.026	0.125

23	1.053	1.049	1.05	34.642	0.014	39.004	0.014	39.911	0.027
24	1.049	1.046	1.048	34.788	0	39.298	0	40.188	0
25	1.06	1.06	1.06	33.725	0.077	37.622	0.132	38.45	0.154
26	1.026	1.026	1.026	33.869	0	37.822	0	38.675	0
27	1.044	1.041	1.043	35.493	0	40.199	-	41.109	0
28	1.04	1.035	1.032	35.742	0.029	40.605	0.061	41.732	0.105
29	1.04	1.035	1.032	35.871	0.016	40.809	0.035	42.071	0.059
30	1.032	1.031	1.031	35.025	0.046	39.461	0.018	40.491	0.019
31	1.042	1.039	1.036	35.813	0	40.71	0	41.941	0
32	1.043	1.041	1.043	35.575	0	40.298	0	41.197	0
33	1.049	1.046	1.045	35.783	0.004	40.514	0.059	41.745	0.059
34	1.056	1.056	1.056	35.508	0.059	40.099	0.008	41.242	0
35	1.053	1.054	1.054	35.592	0.007	40.207	0.006	41.301	0.024
36	1.052	1.054	1.055	35.602	0	40.214	0	41.265	0.019
37	1.06	1.06	1.06	35.383	0.08	39.913	0.035	41.105	0.047
38	1.016	1.015	1.017	35.187	0.028	39.667	0.014	40.906	0.008
39	1.041	1.043	1.042	36.328	0.027	40.828	0.059	42.229	0.059
40	1.037	1.042	1.043	36.648	0	40.987	0	42.343	0
41	1.033	1.036	1.035	36.802	0.035	41.248	0.059	42.807	0.059
42	1.036	1.041	1.04	36.505	0	40.821	0	42.429	0
43	1.05	1.04	1.036	35.896	0.141	40.842	0.007	42.779	0.059
44	1.046	1.032	1.022	35.978	0.227	41.124	0.044	43.866	0.017
45	1.039	1.026	1.019	35.847	0.112	40.949	0.059	43.848	0.059
46	1.041	1.038	1.032	35.256	0	40.045	0	42.777	0
47	1.048	1.045	1.041	34.816	0.026	39.359	0.016	41.828	0.01
48	1.05	1.047	1.042	34.871	0.103	39.403	0.079	41.87	0.028
49	1.05	1.05	1.049	34.599	0	38.958	0	41.221	0
50	1.042	1.039	1.037	35.107	0.014	39.696	0.052	42.152	0.059
51	1.032	1.028	1.025	35.758	0.059	40.654	0.059	43.38	0.059

52	1.028	1.024	1.019	35.988	0.059	41.002	0.059	43.852	0.059
53	1.027	1.023	1.019	36.07	0.059	41.081	0.059	43.856	0.059
54	1.033	1.031	1.031	35.824	0	40.642	0	43.123	0
55	1.033	1.031	1.032	35.879	0	40.643	0	42.948	0.024
56	1.033	1.031	1.031	35.857	0	40.651	0	43.047	0.016
57	1.035	1.032	1.029	35.651	0.002	40.43	0.034	42.941	0.059
58	1.031	1.027	1.024	35.883	0.033	40.788	0.043	43.441	0.059
59	1.047	1.047	1.047	34.739	0	39.319	0	41.815	0
60	1.046	1.046	1.048	34.332	0	38.747	0.003	41.003	0.008
61	1.048	1.048	1.052	34.21	0	38.555	0	40.743	0
62	1.045	1.044	1.047	34.329	0	38.748	0	40.905	0
63	1.017	1.016	1.015	34.498	0.002	38.965	0.004	41.309	0.017
64	1.023	1.023	1.024	34.247	0.027	38.598	0.024	40.792	0.016
65	1.015	1.016	1.017	33.87	0.008	38.021	0.005	39.954	0.005
66	1.06	1.06	1.06	33.731	0.149	37.803	0.212	39.762	0.211
67	1.05	1.047	1.045	34.149	0.059	38.467	0.063	40.63	0.025
68	1.014	1.015	1.016	33.791	0.001	37.949	0.003	39.878	0.007
69	1.06	1.06	1.06	33.539	0	37.576	0	39.353	0
70	1.043	1.039	1.047	35.026	0	39.725	0	40.758	0
71	1.043	1.039	1.049	35.05	0.009	39.761	0.01	40.605	0.009
72	1.044	1.04	1.049	35.05	0	39.744	0	40.374	0
73	1.042	1.038	1.05	35.073	0	39.799	0	40.499	0
74	1.036	1.028	1.033	35.484	0	40.327	0.073	41.465	0.15
75	1.038	1.032	1.037	35.239	0.026	40.027	0.059	41.472	0.059
76	1.032	1.023	1.026	35.6	0.058	40.439	0.164	41.716	0.289
77	1.051	1.048	1.047	34.402	0	38.945	0	40.981	0.009
78	1.048	1.044	1.041	34.459	0.036	39.048	0.064	41.202	0.09
79	1.05	1.046	1.045	34.361	0.028	38.904	0.059	41.092	0.059
80	1.06	1.06	1.06	33.82	0	38.069	0	40.147	0

81	1.01	1.011	1.012	33.791	0.014	37.988	0.016	39.974	0.018
82	1.053	1.044	1.036	34.414	0.091	39.112	0.002	41.649	0.059
83	1.055	1.046	1.037	34.228	0.107	38.89	0.017	41.464	0.059
84	1.055	1.049	1.043	33.774	0.006	38.296	0.059	40.847	0.059
85	1.059	1.055	1.049	33.414	0	37.78	0.037	40.203	0.084
86	1.054	1.048	1.042	33.667	0.03	38.203	0.059	40.868	0.059
87	1.058	1.055	1.049	33.62	0	38.132	0	40.78	0
88	1.055	1.052	1.05	33.098	0.038	37.321	0.059	39.676	0.059
89	1.06	1.06	1.06	32.601	0	36.555	0	38.645	0
90	1.047	1.042	1.04	33.695	0	38.304	0	40.726	0
91	1.05	1.046	1.046	33.634	0	38.14	0	40.337	0
92	1.057	1.054	1.051	33.378	0.022	37.623	0.049	39.83	0.061
93	1.052	1.049	1.046	33.866	0.059	38.271	0.059	40.554	0.059
94	1.051	1.048	1.045	34.176	0.046	38.648	0.059	40.905	0.059
95	1.047	1.043	1.039	34.369	0.059	38.976	0.059	41.418	0.059
96	1.051	1.045	1.041	34.305	0.009	38.887	0.058	41.312	0.059
97	1.053	1.049	1.047	34.126	0.028	38.579	0.059	40.879	0.059
98	1.055	1.052	1.05	34.157	0.017	38.528	0.048	40.641	0.059
99	1.056	1.054	1.056	34.282	0	38.651	0	40.441	0
100	1.059	1.059	1.06	34.142	0	38.358	0	40.08	0
101	1.052	1.051	1.049	33.975	0.059	38.304	0.059	40.357	0.059
102	1.055	1.052	1.05	33.594	0.037	37.878	0.059	40.047	0.059
103	1.052	1.052	1.056	34.885	0	39.124	0	40.44	0
104	1.045	1.043	1.05	35.455	0	39.886	0.004	40.836	0.038
105	1.042	1.041	1.05	35.689	0	40.103	0	40.999	0
106	1.038	1.036	1.043	35.749	0.054	40.214	0.059	41.332	0.059
107	1.033	1.035	1.046	36.347	0	40.58	0	41.283	0
108	1.04	1.04	1.048	35.898	0.001	40.209	0.008	41.068	0.042
109	1.04	1.039	1.047	35.973	0.003	40.235	0.015	41.073	0.059

110	1.04	1.042	1.051	36.065	0	40.141	0	40.854	0.046
111	1.046	1.05	1.06	35.658	0	39.579	0	40.269	0
112	1.029	1.034	1.046	36.877	0	40.729	0	41.36	0
113	1.054	1.055	1.057	35.252	0	39.786	0	40.594	0
114	1.041	1.037	1.036	35.639	0.023	40.418	0.043	41.391	0.059
115	1.04	1.036	1.036	35.642	0.025	40.424	0.047	41.404	0.066
116	1.014	1.015	1.017	33.82	0	37.995	0	39.946	0
117	1.042	1.041	1.037	35.768	0.059	40.678	0.059	42.196	0.059
118	1.032	1.028	1.032	35.527	0.059	40.414	0.059	41.868	0.059

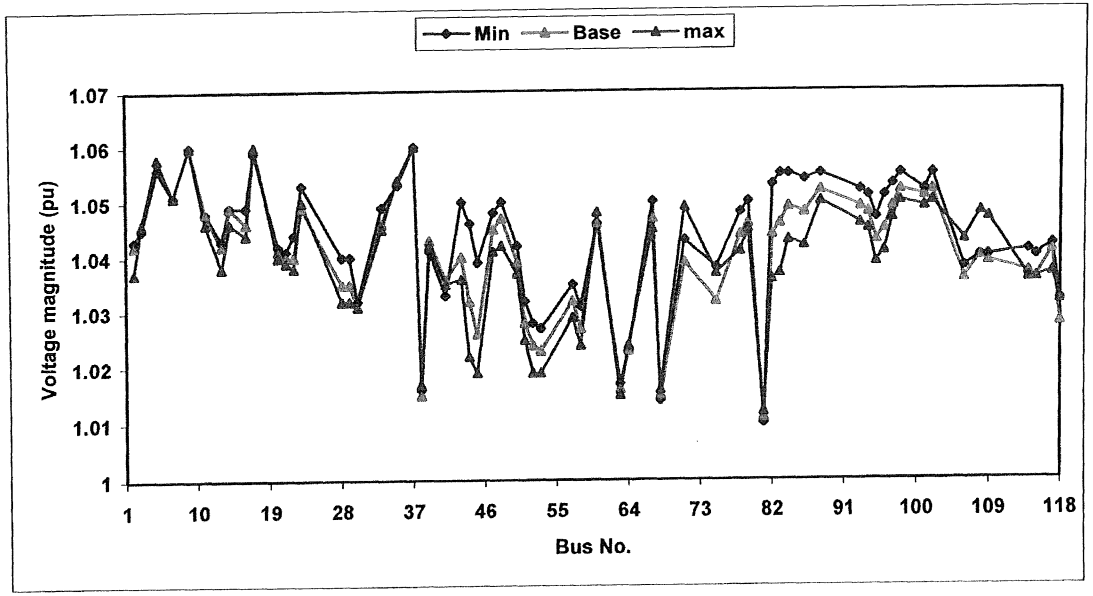


Fig 3.21: Voltage magnitude profiles of IEEE-118 for three load conditions

3.4 Conclusion

In this chapter, a reactive power pricing analysis have been presented in details considering the impact of various factors on reactive power marginal prices i.e., change of objective function and different system operating conditions including load power factor, daily load fluctuation and voltage control. The corresponding optimal power flow problem is defined and a program has been developed for the solution of the OPF problem using constrained non-linear optimization function in MATLAB's Optimization Toolbox [16] with the help of MatPower package [17]. The case studies were carried out on IEEE-14 & IEEE-118 bus systems.

Based on the results obtained on the IEEE-14 bus system and IEEE-118 bus system, the following main conclusions can be drawn:

- Based on various case studies, we conclude that the active power marginal price can be studied independently without considering reactive power production cost.
- The capital investment of capacitors should be considered in reactive power pricing because of their noticeable impacts on reactive power marginal prices.
- Load power factor, daily load fluctuations and bus voltage control and their limits have significant impacts on reactive power marginal prices especially when some system operation limits are reached.
- Reactive power marginal price can serve as a system index related to the urgency of reactive power supply and system voltage support and an incentive to improve load power factor and reduce reactive power demand.
- The revenue based on reactive power marginal price is much higher than that based on reactive power average price. Therefore, some adjustment should be made in using reactive power marginal price.

Chapter 4

Conclusions

4.1 General

This thesis has addressed to optimal reactive power planning and impact of various factors on reactive power marginal pricing using modified optimal power flow. The main contributions of the thesis are the following:

- A modified OPF model consider for reactive power planning with an objective function to minimize the total system operating cost and cost of adding new capacitors, generally consider by other researchers.
- The optimal placement and size of capacitors at load buses are determined from the cost benefit analysis (CBA).
- Impact of change of objective function and other system operating conditions including load power factor, daily load fluctuation and voltage control on reactive power marginal prices have been studied.

The studies were carried out on the IEEE-14 and IEEE-118 bus systems.

4.2 Summary of important Findings

The main findings of the thesis are given below:

In chapter 2, an OPF model formulation for reactive power planning was developed. Constrained non-linear optimization technique was utilized for the solution of the

optimization problem. The test results obtained in this chapter provides the following main conclusions:

- Reactive power supply is essential for reliably operating the electric transmission system. Not only is reactive power necessary to operate the transmission system reliably, but it can also substantially improve the efficiency with which real power is delivered to customers.
- From the utility and economic perspective, installing capacitor units on all load buses to support voltage is not good. With this approach utilities can reduce their capital investment on reactive power sources and same time ensuring the reliable operation of the power system and also get the maximum benefit from the limited reactive power sources.

Chapter 3 has explored the impact of various factors on reactive power marginal prices by considering various system conditions. From the studies conducted on the two systems, the following conclusions can be drawn.

- Results demonstrated that active and reactive power marginal prices give economical signals that could impel even more the participation of agents of competitive reactive power markets.
- It promotes ancillary services markets in deregulated environments.
- RPMPs may be considered an indicator for the installation of new reactive power sources and consequently the bus voltage profile and power quality may be enhanced.
- Active and reactive power marginal prices provide economic signals to reduce reactive power consumption.
- It is observed that a reactive power marginal price is typically less than 1% of the corresponding active power marginal price.
- Due to the known value of active and reactive power marginal prices at each bus, the wheeling charges can be calculated.

- The pricing structure of reactive power cannot only recover the cost of reactive power providers, but also provide economic information for real time operations. These are necessary ingredients for a successful marketplace of electricity.
- Reactive power pricing encourages efficient locational siting of new generation. New generation might choose locations that reduce system reactive power needs because reactive power losses in transmission lines are very high, generators near loads can supply reactive power with much lower losses than generators located long distances from loads.

4.3 Scope for Future Research

As a consequence of the investigations carried out in this thesis, the following works are identified for future work in this area.

- The nodal price varies in both space and time and is composed of the variable operation costs and any additional charges for maintaining quality and reliable electricity services. Lagrangian multipliers cannot give a detailed description of each nodal price, which is demanded by the power industry. With the help decomposition techniques, breaking down each nodal price into a variety of parts corresponding to the concerned factors, such as generations, transmission congestion, voltage limitations and other constraints or elements. This full information for nodal prices can be used not only to improve the efficient usage of power grid and congestion management, but also to design a reasonable pricing structure of power systems, or to provide economic signals for generation or transmission investment.

- The cost of reactive power production modeling is difficult because of differences in reactive power generation equipments, local geographical characteristics of reactive power, and the relationship between voltage and reactive power support. The present study has considered only reactive power production cost of static compensators. We can also include a detail modeling of the synchronous generator (dynamic reactive power sources) by use of nonlinear model that

represents loss of opportunity in commercialization of active power. Detail modeling includes modeling of heating limits of armature field, and of the under-excitation limit of the cylindrical rotor generator.

- Solving the modified optimal power flow for large power systems may not be advisable owing to local characteristics of the reactive power pricing. Therefore reactive power reliability needs should be assessed locally based on this principle, an alternative method is, a given power system can be separated into some non overlapping voltage-control areas comprising coherent bus groups. A set of buses can be classified as a voltage-control area if they are sufficiently uncoupled electrically, from its neighboring areas. And the controllable reactive power in the area should be enough to master the voltage changes at the buses in the area. The controls of each area are much less influenced by other areas. For each area a MOPF problem would be separately solved. Therefore, it will enhance the computation speed. It also helps in creating efficient competitive ancillary services markets within the regions and possible of gaming will be avoided.

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Appendix A

Data for IEEE-14 Bus System

(At 100 MVA Base)

The IEEE-14 bus system is shown in Fig. A.1. The system data is taken from [18]. The relevant data are shown in below tables.

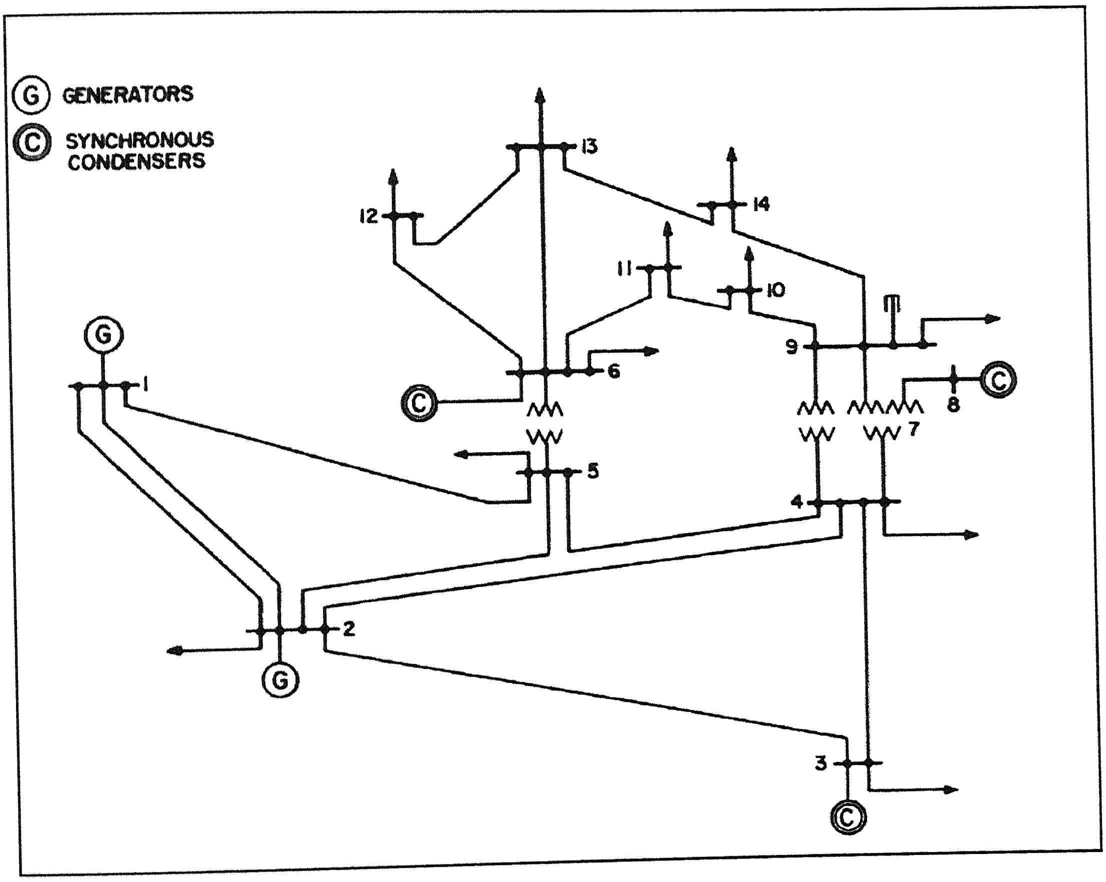

Fig. A.1: IEEE-14 bus system

Table A.1: IEEE-14 -Bus data (in p.u)

Bus No.	V_m	P_g	Q_g	P_d	Q_d	Q_{max}	Q_{min}	P_{max}	P_{min}	External Shunt Susceptance
1	1.06	2.324	-0.169	0	0	0.1	0	3.324	0	0
2	1.045	0.4	0.424	0.217	0.127	0.5	-0.4	1.4	0	0
3	1.01	0	0.234	0.942	0.19	0.4	0	1	0	0
4	1.019	0	0	0.478	-0.039	-	-	-	-	0
5	1.02	0	0	0.076	0.016	-	-	-	-	0
6	1.07	0	0.122	0.112	0.075	0.24	-0.06	1	0	0
7	1.062	0	0	0	0	-	-	-	-	0
8	1.09	0	0.174	0	0	0.24	-0.06	1	0	0
9	1.056	0	0	0.295	0.166	-	-	-	-	0.19
10	1.051	0	0	0.09	0.058	-	-	-	-	0
11	1.057	0	0	0.035	0.018	-	-	-	-	0
12	1.055	0	0	0.061	0.016	-	-	-	-	0
13	1.05	0	0	0.135	0.058	-	-	-	-	0
14	1.036	0	0	0.149	0.05	-	-	-	-	0

Table A.2: IEEE-14- Line data (in p.u)

Line No.	From	To	R	X	B_{sh} (full)
1	1	2	0.01938	0.05917	0.0528
2	1	5	0.05403	0.22304	0.0492
3	2	3	0.04699	0.19797	0.0438
4	2	4	0.05811	0.17632	0.034
5	2	5	0.05695	0.17388	0.0346
6	3	4	0.06701	0.17103	0.0128
7	4	5	0.01335	0.04211	0
8	6	11	0.09498	0.1989	0
9	6	12	0.12291	0.25581	0
10	6	13	0.06615	0.13027	0
11	7	8	0	0.17615	0
12	7	9	0	0.11001	0
13	9	10	0.03181	0.0845	0
14	9	14	0.12711	0.27038	0
15	10	11	0.08205	0.19207	0
16	12	13	0.22092	0.19988	0
17	13	14	0.17093	0.34802	0

Table A.3: IEEE-14 bus -Transformer data (in p.u)

Line No.	From	To	R	X	Tap ratio
1	4	7	0	0.20912	0.978
2	4	9	0	0.55618	0.969
3	5	6	0	0.25202	0.932

Table A.4: IEEE-14-Generator cost characteristics

Bus No.	a ($/MW^2-hr)	b (S/MW-hr)	c ($/Hr)
1	0.043029	20	0
2	0.25	20	0
3	0.01	40	0
6	0.01	40	0
8	0.01	40	0

Appendix B

Data for IEEE-118 Bus System

(At 100 MVA Base)

The IEEE-118 bus system is shown in Fig. B.1. The system data is taken from [18]. The relevant data are provided in following tables.

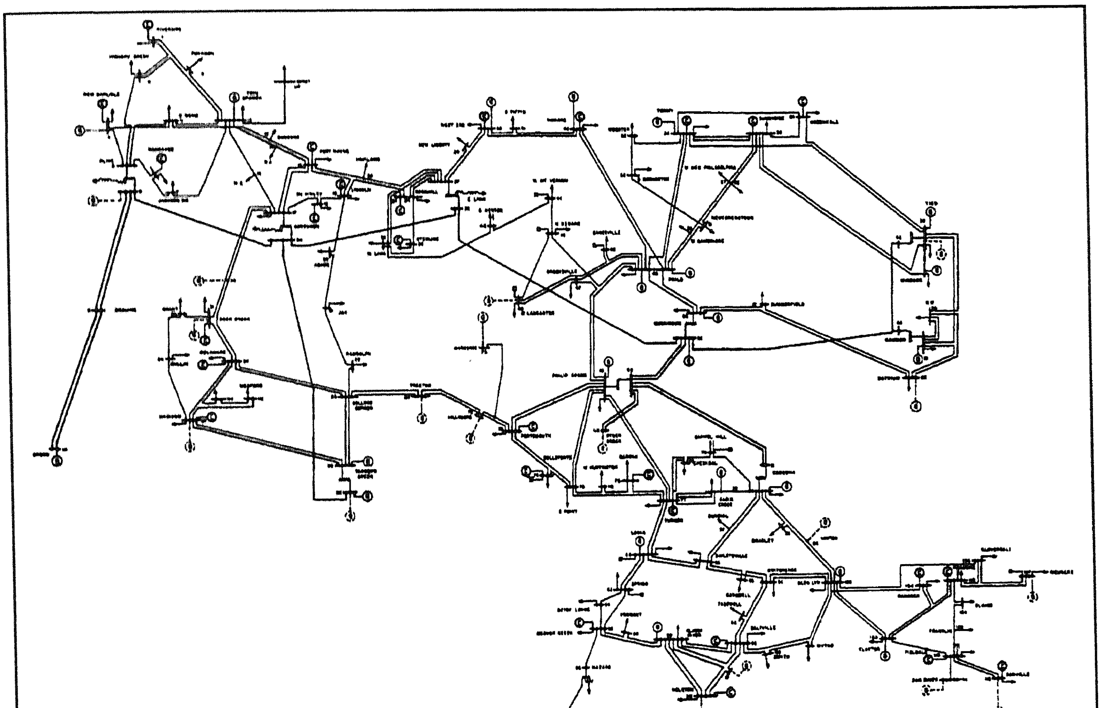

Fig. B.1: IEEE-118 bus system

Table B.1: IEEE-118-Bus data (in p.u)

Bus No.	Vm	Pg	Qg	Pd	Qd	Qmax	Qmin	Pmax	Pmin	External shunt Susceptance
1	0.955	0	0	0.51	0.27	0.15	-0.05	1	0	0
2	0.971	-	-	0.2	0.09	-	-	-	-	0
3	0.968			0.39	0.1	-	-	-	-	0
4	0.998	0	0	0.39	0.12	3	-3	1	0	0
5	1.002	-	-	0	0	-	-	-	-	-0.4
6	0.99	0	0	0.52	0.22	0.5	-0.13	1	0	0
7	0.989	-	-	0.19	0.02	-	-	-	-	0
8	1.015	0	0	0.28	0	3	-3	1	0	0
9	1.043	-	-	0	0	-	-	-	-	0
10	1.05	4.5	0	0	0	2	-1.47	5.5	0	0
11	0.985	-	-	0.7	0.23	-	-	-	-	0
12	0.99	0.85	0	0.47	0.1	1.2	-0.35	1.85	0	0
13	0.968	-	-	0.34	0.16	-	-	-	-	0
14	0.984	-	-	0.14	0.01	-	-	-	-	0
15	0.97	0	0	0.9	0.3	0.3	-0.1	1	0	0
16	0.984	-	-	0.25	0.1	-	-	-	-	0
17	0.995	-	-	0.11	0.03	-	-	-	-	0
18	0.973	0	0	0.6	0.34	0.5	-0.16	1	0	0
19	0.963	0	0	0.45	0.25	0.24	-0.08	1	0	0
20	0.958	-	-	0.18	0.03	-	-	-	-	0
21	0.959	-	-	0.14	0.08	-	-	-	-	0
22	0.97	-	-	0.1	0.05	-	-	-	-	0
23	1	-	-	0.07	0.03	-	-	-	-	0
24	0.992	0	0	0.13	0	3	-3	1	0	0
25	1.05	2.2	0	0	0	1.4	-0.47	3.2	0	0
26	1.015	3.14	0	0	0	10	-10	4.14	0	0
27	0.968	0	0	0.71	0.13	3	-3	1	0	0
28	0.962	-	-	0.17	0.07	-	-	-	-	0
29	0.963	-	-	0.24	0.04	-	-	-	-	0
30	0.968	-	-	0	0	-	-	-	-	0
31	0.967	0.07	0	0.43	0.27	3	-3	1.07	0	0
32	0.964	0	0	0.59	0.23	0.42	-0.42	1	0	0
33	0.972	-	-	0.23	0.09	-	-	-	-	0
34	0.986	0	0	0.59	0.26	0.24	-0.08	1	0	0.14
35	0.981	-	-	0.33	0.09	-	-	-	-	0
36	0.98	0	0	0.31	0.17	0.24	-0.08	1	0	0
37	0.992	-	-	0	0	-	-	-	-	-0.25
38	0.962	-	-	0	0	-	-	-	-	0
39	0.97	-	-	0.27	0.11	-	-	-	-	0

40	0.97	0	0	0.66	0.23	3	-3	1	0	0
41	0.967	-	-	0.37	0.1	-	-	-	-	0
42	0.985	0	0	0.96	0.23	3	-3	1	0	0
43	0.978	-	-	0.18	0.07	-	-	-	-	0
44	0.985	-	-	0.16	0.08	-	-	-	-	0.1
45	0.987	-	-	0.53	0.22	-	-	-	-	0.1
46	1.005	0.19	0	0.28	0.1	1	-1	1.19	0	0.1
47	1.017	-	-	0.34	0	-	-	-	-	0
48	1.021	-	-	0.2	0.11	-	-	-	-	0.15
49	1.025	2.04	0	0.87	0.3	2.1	-0.85	3.04	0	0
50	1.001	-	-	0.17	0.04	-	-	-	-	0
51	0.967	-	-	0.17	0.08	-	-	-	-	0
52	0.957	-	-	0.18	0.05	-	-	-	-	0
53	0.946	-	-	0.23	0.11	-	-	-	-	0
54	0.955	0.48	0	1.13	0.32	3	-3	1.48	0	0
55	0.952	0	0	0.63	0.22	0.23	-0.08	1	0	0
56	0.954	0	0	0.84	0.18	0.15	-0.08	1	0	0
57	0.971	-	-	0.12	0.03	-	-	-	-	0
58	0.959	-	-	0.12	0.03	-	-	-	-	0
59	0.985	1.55	0	2.77	1.13	1.8	-0.6	2.55	0	0
60	0.993	-	-	0.78	0.03	-	-	-	-	0
61	0.995	1.6	0	0	0	3	-1	2.6	0	0
62	0.998	0	0	0.77	0.14	0.2	-0.2	1	0	0
63	0.969	-	-	0	0	-	-	-	-	0
64	0.984	-	-	0	0	-	-	-	-	0
65	1.005	3.91	0	0	0	2	-0.67	4.91	0	0
66	1.05	3.92	0	0.39	0.18	2	-0.67	4.92	0	0
67	1.02	-	-	0.28	0.07	-	-	-	-	0
68	1.003	-	-	0	0	-	-	-	-	0
69	1.035	5.164	0	0	0	3	-3	8.052	0	0
70	0.984	0	0	0.66	0.2	0.32	-0.1	1	0	0
71	0.987	-	-	0	0	-	-	-	-	0
72	0.98	0	0	0.12	0	1	-1	1	0	0
73	0.991	0	0	0.06	0	1	-1	1	0	0
74	0.958	0	0	0.68	0.27	0.09	-0.06	1	0	0.12
75	0.967	-	-	0.47	0.11	-	--	-	-	0
76	0.943	0	0	0.68	0.36	0.23	-0.08	1	0	0
77	1.006	0	0	0.61	0.28	0.7	-0.2	1	0	0
78	1.003	-	-	0.71	0.26	-	-	-	-	0
79	1.009	-	-	0.39	0.32	-	-	-	-	0.2
80	1.04	4.77	0	1.3	0.26	2.8	-1.65	5.77	0	0
81	0.997	-	-	0	0	-	-	-	-	0
82	0.989	-	-	0.54	0.27	-	-	-	-	0.2
83	0.985	-	-	0.2	0.1	-	-	-	-	0.1

84	0.98	-	-	0.11	0.07	-	-	-	-	0
85	0.985	0	0	0.24	0.15	0.23	-0.08	1	0	0
86	0.987	-	-	0.21	0.1	-	-	-	-	0
87	1.015	0.04	0	0	0	10	-1	1.04	0	0
88	0.987	-	-	0.48	0.1	-	-	-	-	0
89	1.005	6.07	0	0	0	3	-2.1	7.07	0	0
90	0.985	0	0	1.63	0.42	3	-3	1	0	0
91	0.98	0	0	0.1	0	1	-1	1	0	0
92	0.993	0	0	0.65	0.1	0.09	-0.03	1	0	0
93	0.987	-	-	0.12	0.07	-	-	-	-	0
94	0.991	-	-	0.3	0.16	-	-	-	-	0
95	0.981	-	-	0.42	0.31	-	-	-	-	0
96	0.993	-	-	0.38	0.15	-	-	-	-	0
97	1.011	-	-	0.15	0.09	-	-	-	-	0
98	1.024	-	-	0.34	0.08	-	-	-	-	0
99	1.01	0	0	0.42	0	1	-1	1	0	0
100	1.017	2.52	0	0.37	0.18	1.55	-0.5	3.52	0	0
101	0.993	-	-	0.22	0.15	-	-	-	-	0
102	0.991	-	-	0.05	0.03	-	-	-	-	0
103	1.001	0.4	0	0.23	0.16	0.4	-0.15	1.4	0	0
104	0.971	0	0	0.38	0.25	0.23	-0.08	1	0	0
105	0.965	0	0	0.31	0.26	0.23	-0.08	1	0	0.2
106	0.962	-	-	0.43	0.16	-	-	-	-	0
107	0.952	0	0	0.5	0.12	2	-2	1	0	0.06
108	0.967	-	-	0.02	0.01	-	-	-	-	0
109	0.967	-	-	0.08	0.03	-	-	-	-	0
110	0.973	0	0	0.39	0.3	0.23	-0.08	1	0	0.06
111	0.98	0.36	0	0	0	10	-1	1.36	0	0
112	0.975	0	0	0.68	0.13	10	-1	1	0	0
113	0.993	0	0	0.06	0	2	-1	1	0	0
114	0.96	-	-	0.08	0.03	-	-	-	-	0
115	0.96	-	-	0.22	0.07	-	-	-	-	0
116	1.005	0	0	1.84	0	10	-10	1	0	0
117	0.974	-	-	0.2	0.08	-	-	-	-	0
118	0.949	-	-	0.33	0.15	-	-	-	-	0

Table B.2: IEEE-118 bus-Line data (in p.u)

Line No.	From	To	R	X	B_{sh} (full)
1	1	2	0.0303	0.0999	0.0254
2	1	3	0.0129	0.0424	0.01082
3	4	5	0.00176	0.00798	0.0021
4	3	5	0.0241	0.108	0.0284

5	5	6	0.0119	0.054	0.01426
6	6	7	0.00459	0.0208	0.0055
7	8	9	0.00244	0.0305	1.162
8	9	10	0.00258	0.0322	1.23
9	4	11	0.0209	0.0688	0.01748
10	5	11	0.0203	0.0682	0.01738
11	11	12	0.00595	0.0196	0.00502
12	2	12	0.0187	0.0616	0.01572
13	3	12	0.0484	0.16	0.0406
14	7	12	0.00862	0.034	0.00874
15	11	13	0.02225	0.0731	0.01876
16	12	14	0.0215	0.0707	0.01816
17	13	15	0.0744	0.2444	0.06268
18	14	15	0.0595	0.195	0.0502
19	12	16	0.0212	0.0834	0.0214
20	15	17	0.0132	0.0437	0.0444
21	16	17	0.0454	0.1801	0.0466
22	17	18	0.0123	0.0505	0.01298
23	18	19	0.01119	0.0493	0.01142
24	19	20	0.0252	0.117	0.0298
25	15	19	0.012	0.0394	0.0101
26	20	21	0.0183	0.0849	0.0216
27	21	22	0.0209	0.097	0.0246
28	22	23	0.0342	0.159	0.0404
29	23	24	0.0135	0.0492	0.0498
30	23	25	0.0156	0.08	0.0864
31	25	27	0.0318	0.163	0.1764
32	27	28	0.01913	0.0855	0.0216
33	28	29	0.0237	0.0943	0.0238
34	8	30	0.00431	0.0504	0.514
35	26	30	0.00799	0.086	0.908
36	17	31	0.0474	0.1563	0.0399
37	29	31	0.0108	0.0331	0.0083
38	23	32	0.0317	0.1153	0.1173
39	31	32	0.0298	0.0985	0.0251
40	27	32	0.0229	0.0755	0.01926
41	15	33	0.038	0.1244	0.03194
42	19	34	0.0752	0.247	0.0632
43	35	36	0.00224	0.0102	0.00268
44	35	37	0.011	0.0497	0.01318
45	33	37	0.0415	0.142	0.0366
46	34	36	0.00871	0.0268	0.00568
47	34	37	0.00256	0.0094	0.00984
48	37	39	0.0321	0.106	0.027

49	37	40	0.0593	0.168	0.042
50	30	38	0.00464	0.054	0.422
51	39	40	0.0184	0.0605	0.01552
52	40	41	0.0145	0.0487	0.01222
53	40	42	0.0555	0.183	0.0466
54	41	42	0.041	0.135	0.0344
55	43	44	0.0608	0.2454	0.06068
56	34	43	0.0413	0.1681	0.04226
57	44	45	0.0224	0.0901	0.0224
58	45	46	0.04	0.1356	0.0332
59	46	47	0.038	0.127	0.0316
60	46	48	0.0601	0.189	0.0472
61	47	49	0.0191	0.0625	0.01604
62	42	49	0.0715	0.323	0.086
63	42	49	0.0715	0.323	0.086
64	45	49	0.0684	0.186	0.0444
65	48	49	0.0179	0.0505	0.01258
66	49	50	0.0267	0.0752	0.01874
67	49	51	0.0486	0.137	0.0342
68	51	52	0.0203	0.0588	0.01396
69	52	53	0.0405	0.1635	0.04058
70	53	54	0.0263	0.122	0.031
71	49	54	0.073	0.289	0.0738
72	49	54	0.0869	0.291	0.073
73	54	55	0.0169	0.0707	0.0202
74	54	56	0.00275	0.00955	0.00732
75	55	56	0.00488	0.0151	0.00374
76	56	57	0.0343	0.0966	0.0242
77	50	57	0.0474	0.134	0.0332
78	56	58	0.0343	0.0966	0.0242
79	51	58	0.0255	0.0719	0.01788
80	54	59	0.0503	0.2293	0.0598
81	56	59	0.0825	0.251	0.0569
82	56	59	0.0803	0.239	0.0536
83	55	59	0.04739	0.2158	0.05646
84	59	60	0.0317	0.145	0.0376
85	59	61	0.0328	0.15	0.0388
86	60	61	0.00264	0.0135	0.01456
87	60	62	0.0123	0.0561	0.01468
88	61	62	0.00824	0.0376	0.0098
89	63	64	0.00172	0.02	0.216
90	38	65	0.00901	0.0986	1.046
91	64	65	0.00269	0.0302	0.38
92	49	66	0.018	0.0919	0.0248

93	49	66	0.018	0.0919	0.0248
94	62	66	0.0482	0.218	0.0578
95	62	67	0.0258	0.117	0.031
96	66	67	0.0224	0.1015	0.02682
97	65	68	0.00138	0.016	0.638
98	47	69	0.0844	0.2778	0.07092
99	49	69	0.0985	0.324	0.0828
100	69	70	0.03	0.127	0.122
101	24	70	0.00221	0.4115	0.10198
102	70	71	0.00882	0.0355	0.00878
103	24	72	0.0488	0.196	0.0488
104	71	72	0.0446	0.18	0.04444
105	71	73	0.00866	0.0454	0.01178
106	70	74	0.0401	0.1323	0.03368
107	70	75	0.0428	0.141	0.036
108	69	75	0.0405	0.122	0.124
109	74	75	0.0123	0.0406	0.01034
110	76	77	0.0444	0.148	0.0368
111	69	77	0.0309	0.101	0.1038
112	75	77	0.0601	0.1999	0.04978
113	77	78	0.00376	0.0124	0.01264
114	78	79	0.00546	0.0244	0.00648
115	77	80	0.017	0.0485	0.0472
116	77	80	0.0294	0.105	0.0228
117	79	80	0.0156	0.0704	0.0187
118	68	81	0.00175	0.0202	0.808
119	77	82	0.0298	0.0853	0.08174
120	82	83	0.0112	0.03665	0.03796
121	83	84	0.0625	0.132	0.0258
122	83	85	0.043	0.148	0.0348
123	84	85	0.0302	0.0641	0.01234
124	85	86	0.035	0.123	0.0276
125	86	87	0.02828	0.2074	0.0445
126	85	88	0.02	0.102	0.0276
127	85	89	0.0239	0.173	0.047
128	88	89	0.0139	0.0712	0.01934
129	89	90	0.0518	0.188	0.0528
130	89	90	0.0238	0.0997	0.106
131	90	91	0.0254	0.0836	0.0214
132	89	92	0.0099	0.0505	0.0548
133	89	92	0.0393	0.1581	0.0414
134	91	92	0.0387	0.1272	0.03268
135	92	93	0.0258	0.0848	0.0218
136	92	94	0.0481	0.158	0.0406

137	93	94	0.0223	0.0732	0.01876
138	94	95	0.0132	0.0434	0.0111
139	80	96	0.0356	0.182	0.0494
140	82	96	0.0162	0.053	0.0544
141	94	96	0.0269	0.0869	0.023
142	80	97	0.0183	0.0934	0.0254
143	80	98	0.0238	0.108	0.0286
144	80	99	0.0454	0.206	0.0546
145	92	100	0.0648	0.295	0.0472
146	94	100	0.0178	0.058	0.0604
147	95	96	0.0171	0.0547	0.01474
148	96	97	0.0173	0.0885	0.024
149	98	100	0.0397	0.179	0.0476
150	99	100	0.018	0.0813	0.0216
151	100	101	0.0277	0.1262	0.0328
152	92	102	0.0123	0.0559	0.01464
153	101	102	0.0246	0.112	0.0294
154	100	103	0.016	0.0525	0.0536
155	100	104	0.0451	0.204	0.0541
156	103	104	0.0466	0.1584	0.0407
157	103	105	0.0535	0.1625	0.0408
158	100	106	0.0605	0.229	0.062
159	104	105	0.00994	0.0378	0.00986
160	105	106	0.014	0.0547	0.01434
161	105	107	0.053	0.183	0.0472
162	105	108	0.0261	0.0703	0.01844
163	106	107	0.053	0.183	0.0472
164	108	109	0.0105	0.0288	0.0076
165	103	110	0.03906	0.1813	0.0461
166	109	110	0.0278	0.0762	0.0202
167	110	111	0.022	0.0755	0.02
168	110	112	0.0247	0.064	0.062
169	17	113	0.00913	0.0301	0.00768
170	32	113	0.0615	0.203	0.0518
171	32	114	0.0135	0.0612	0.01628
172	27	115	0.0164	0.0741	0.01972
173	114	115	0.0023	0.0104	0.00276
174	68	116	0.00034	0.00405	0.164
175	12	117	0.0329	0.14	0.0358
176	75	118	0.0145	0.0481	0.01198
177	76	118	0.0164	0.0544	0.01356

Table B.3: IEEE-118 bus-Transformer data (in p.u)

Line No.	From	To	R	X	Tap ratio
1	8	5	0	0.0267	0.985
2	26	25	0	0.0382	0.96
3	30	17	0	0.0388	0.96
4	38	37	0	0.0375	0.935
5	63	59	0	0.0386	0.96
6	64	61	0	0.0268	0.985
7	65	66	0	0.037	0.935
8	68	69	0	0.037	0.935
9	81	80	0	0.037	0.935

Table B.4: IEEE-118 bus-Generator cost characteristics

Bus No.	a ($/MW^2-hr)	b (S/MW-hr)	c ($/Hr)
1	0.01	40	0
4	0.01	40	0
6	0.01	40	0
8	0.01	40	0
10	0.022222	20	0
12	0.117647	20	0
15	0.01	40	0
18	0.01	40	0
19	0.01	40	0
24	0.01	40	0
25	0.045455	20	0
26	0.031847	20	0
27	0.01	40	0
31	1.42857	20	0
32	0.01	40	0
34	0.01	40	0
36	0.01	40	0
40	0.01	40	0
42	0.01	40	0
46	0.526316	20	0
49	0.04902	20	0
54	0.208333	20	0
55	0.01	40	0
56	0.01	40	0
59	0.064516	20	0
61	0.0625	20	0
62	0.01	40	0

65	0.025575	20	0
66	0.02551	20	0
69	0.019365	20	0
70	0.01	40	0
72	0.01	40	0
73	0.01	40	0
74	0.01	40	0
76	0.01	40	0
77	0.01	40	0
80	0.020964	20	0
85	0.01	40	0
87	2.5	20	0
89	0.016475	20	0
90	0.01	40	0
91	0.01	40	0
92	0.01	40	0
99	0.01	40	0
100	0.039683	20	0
103	0.25	20	0
104	0.01	40	0
105	0.01	40	0
107	0.01	40	0
110	0.01	40	0
111	0.277778	20	0
112	0.01	40	0
113	0.01	40	0
116	0.01	40	0

Appendix C

Cost of static compensation equipment

The production cost of any reactive power compensation equipment must include the capital investment return, which is expressed through a depreciation rate depending on its life time [30]. For present studies, static compensator with an initial cost of \$11,600/MVAr, lifetime of 30 years and average use of ¾, hence the cost of reactive power production calculated as

$$\begin{aligned} CAPCOST &= Q_c \bullet \frac{11600}{30 \bullet 365 \bullet 24 \bullet 3/4} \\ &= Q_c \bullet 0.0589 \bullet \frac{\$}{MVAr - Hr} \\ &= Q_c \bullet 1.4136 \bullet \frac{\$}{MVAr - Day} \end{aligned} \tag{C.1}$$

Where,

Q_c is the reactive power generated by the equipment. The impact of the capacitor capital investment in the reactive power cost is represented in [C.1].

Appendix D

Generator supplying Power to a large system

Assume that a generator supplying power to a large system under stable conditions is as shown in Fig. D.1.

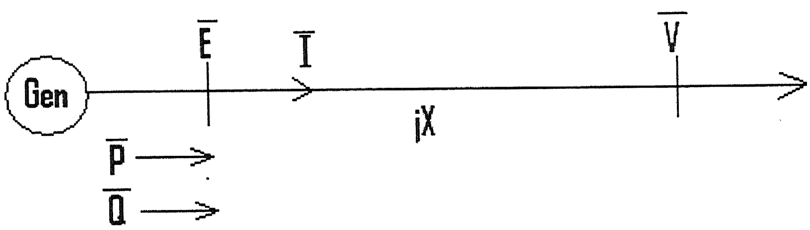

Fig. D.1: Generator supplying power to a large system.

Where

V: Voltage magnitude at the system bus

E: Voltage magnitude at the generator terminal bus

δ: Rotor angle

Consider,

$V = |V|\angle 0^o$ and $E = |E|\angle\delta$

$$I = \frac{|E|\angle\delta - |V|}{jX} \tag{D.1}$$

and

$$I^* = \frac{|E|\angle -\delta \;-\; |V|}{-jX} \tag{D.2}$$

Therefore,

$$
\begin{aligned}
P + jQ &= V\ I^* \\
&= \frac{|V|\bullet|E|\angle -\delta \;-\; |V|^2}{-jX} \\
&= \frac{|V|\bullet|E|\angle 90-\delta \;-\; |V|^2\angle 90^o}{X}
\end{aligned} \tag{D.3}
$$

The real part of the equation (D.3) is

$$P = \frac{|V|\bullet|E|}{X}\cos(90-\delta) = \frac{|V|\bullet|E|}{X}\sin\delta \tag{D.4}$$

and the imaginary part of the equation (D.3) is

$$
\begin{aligned}
Q &= \frac{|V|\bullet|E|}{X}\sin(90-\delta) - \frac{|V|^2}{X} \\
&= \frac{|V|}{X}\left(|E|\cos\delta - |V|\right)
\end{aligned} \tag{D.5}
$$